Clifford Algebra

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \qquad \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$$

$$\{\gamma^\mu, \gamma_5\} = 0 \quad \leftarrow \text{leads to inconsistencies in naive dim. reg.}$$

Gamma-matrix contraction identities (Chisholm id.)

① $\gamma^\mu\gamma_\mu = \frac{1}{2}\{\gamma^\mu, \gamma_\mu\} = \frac{1}{2}\, 2g^\mu{}_\mu\, \mathbb{1} = d\cdot\mathbb{1}$ $\qquad g^\rho{}_\rho = d = 4$

② $\gamma^\mu\gamma^\nu\gamma_\mu = -\gamma^\nu\gamma^\mu\gamma_\mu + 2g^{\mu\nu}\gamma_\mu$

$= -\gamma^\nu d + 2\gamma^\nu$

$= (-d+2)\gamma^\nu.$

③ $\gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu = -\gamma^\nu\gamma^\mu\gamma^\rho\gamma_\mu + 2g^{\mu\nu}\gamma^\rho\gamma_\mu$

$= -\gamma^\nu(-d+2)\gamma^\rho + 2\gamma^\rho\gamma^\nu$

$= +d\,\gamma^\nu\gamma^\rho - 2(\gamma^\nu\gamma^\rho - \gamma^\rho\gamma^\nu)$

$= +d\,\gamma^\nu\gamma^\rho - 2(2\gamma^\nu\gamma^\rho - 2g^{\nu\rho})$

$= 4g^{\nu\rho}\mathbb{1} + (d-4)\gamma^\nu\gamma^\rho$

Generally: push γ^μ all the way to the right.

$$\gamma^\mu \underbrace{\gamma^{\nu_1}\gamma^{\nu_2}\ldots\gamma^{\nu_n}}_{\text{string of } n \text{ gammas}}\gamma_\mu = (-1)^n\gamma^{\nu_1}\gamma^{\nu_2}\ldots\gamma^{\nu_N}\underbrace{\gamma^\mu\gamma_\mu}_{d} + \sum_{m=1}^{n}(-1)^{m-1}\, 2g^{\mu\nu_m}\,\gamma^{\nu_1}\ldots\hat{\gamma}^{\nu_m}\ldots\gamma^{\nu_n}\gamma_\mu$$

$$= (-1)^n d\,\gamma^{\nu_1}\ldots\gamma^{\nu_N} + \sum_{m=1}^{n}(-1)^{m-1}\, 2\gamma^{\nu_1}\ldots\hat{\gamma}^{\nu_m}\ldots\gamma^{\nu_n}\gamma^{\nu_m}$$

(essentially last gamma matrix becomes γ^{ν_m})

To normal order gamma matrices

$$13524 = -13254 + 13\overbrace{52}4$$

$$= +12354 - 1\overbrace{32}54 + 1\overbrace{352}4$$

$$= -12345 + 1\overbrace{2354} + 1\overbrace{3245} - 1\overbrace{3254} + 1\overbrace{345}2$$

✓ ✓ ✓ ✓ ✓

Wick's theorem: only contract those that are out of order

$$13524 = \ominus :13524: + :1\overbrace{352}4: + :1\overbrace{35}24: + :1\overbrace{3524}: + :13\overbrace{52}4: \ominus :13\overbrace{52}4: + :13524:$$

$$\gamma^{\mu_1}\gamma^{\mu_3}\gamma^{\mu_5}\gamma^{\mu_2}\gamma^{\mu_4} = -\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}\gamma^{\mu_5} + \gamma^{\mu_5}\gamma^{\mu_5}\gamma^{\mu_4} g^{\mu_3\mu_2} + \gamma^{\mu_1}\gamma^{\mu_3}\gamma^{\mu_4} g^{\mu_2\mu_5} + \gamma^{\mu_1}\gamma^{\mu_3}\gamma^{\mu_2} g^{\mu_4\mu_5} - \gamma^{\mu_1} g^{\mu_2\mu_3} g^{\mu_4\mu_5}$$