Basis of sixteen 4x4 Hermitian matrices (definite parity)

Sixteen linearly independent Hermitian 4x4 matrices – (non-chiral basis) organized into 5 classes: Scalar, Vector, Tensor, Axial vector, Pseudoscalar.

Contravariant Γ^A

$\Gamma^A_{(S)} = \mathbb{1}$ $\qquad A = \{1\}$ $\qquad \mathbb{1} = \Gamma^1$

$\Gamma^A_{(V)} = \gamma^\mu$ $\qquad = \{2,3,4,5\}$ $\qquad \gamma^\mu = (\Gamma^2, \Gamma^3, \Gamma^4, \Gamma^5)$

$\Gamma^A_{(T)} = \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$ $\qquad = \{6,7,8,9,10,11\}$ $\qquad \sigma^{\mu\nu} = \begin{pmatrix} 0 & \Gamma^6 & \Gamma^7 & \Gamma^8 \\ & 0 & \Gamma^{11} & -\Gamma^{10} \\ & & 0 & \Gamma^9 \\ & & & 0 \end{pmatrix}$

$\Gamma^A_{(A)} = \gamma^\mu \gamma_5$ $\qquad = \{12, 13, 14, 15\}$ $\qquad \gamma^\mu\gamma_5 = (\Gamma^{12}, \Gamma^{13}, \Gamma^{14}, \Gamma^{15})$

$\Gamma^A_{(S)} = i\gamma_5$ $\qquad = \{16\}$ $\qquad i\gamma_5 = \Gamma^{16}$

Covariant $\Gamma_A = (\Gamma^A)^{-1}$

S: $\mathbb{1}^{-1} = \mathbb{1}$ $\qquad \Gamma_{(S)A} = \mathbb{1}$

V: $(\gamma^0)^{-1} = \gamma^0$ $\qquad \Gamma_{(V)A} = \gamma_\mu$

$(\vec{\gamma})^{-1} = -\vec{\gamma}$

T: $(\sigma_E^{\mu\nu})^{-1} = -\sigma_E^{\mu\nu}$ $\qquad \Gamma_{(T)A} = \sigma_{\mu\nu}$

$(\sigma_B^{\mu\nu})^{-1} = \sigma_B^{\mu\nu}$

A: $(\gamma^0\gamma_5)^{-1} = -\gamma_0\gamma_5$ $\qquad \Gamma_{(A)A} \overset{(!)}{=} -\gamma_\mu\gamma_5 = \gamma_5\gamma_\mu$

$(\vec{\gamma}\gamma_5)^{-1} = \vec{\gamma}\gamma_5$

P: $(\gamma_5)^{-1} = -i\gamma_5$ $\qquad \Gamma_{(P)A} = -i\gamma_5$

Orthonormality Relations

$$\mathrm{Tr}[\Gamma_A \Gamma^B] = 4\delta_A^{\ B}$$

Dirac Hermiticity

$$(\Gamma^A)^\dagger = \gamma^0 \Gamma^A \gamma^0$$