Chisholm expansion (Englishman: John Stephen Roy Chisholm 1926–2015)

Any 4×4 hermitian matrix S can be resolved into a sum of 16 Dirac matrices:

$$S = \sum_{A=1}^{16} a_A \Gamma^A$$

To avoid double counting.

$$= a_0 \mathbb{1} + a_\mu \gamma^\mu + \frac{1}{2} a_{\mu\nu}\sigma^{\mu\nu} + a_{5\mu}\gamma^\mu\gamma_5 + a_5(i\gamma_5) \qquad (*)$$

c.f: this is analogous to the Pauli matrix expansion

$$\mathbb{A} = c_0 \mathbb{1}_{2\times 2} + c_1\sigma^1 + c_2\sigma^2 + c_3\sigma^3 = c_0\mathbb{1} + \vec{c}\cdot\vec{\sigma}$$

Products of gamma matrices themselves can be resolved into the basis:

e.g: $\gamma^\mu\gamma^\nu = g^{\mu\nu}\mathbb{1} - i\sigma^{\mu\nu}$

$$\gamma^\mu\gamma^\nu\gamma^\rho = g^{\mu\nu}\gamma^\rho - g^{\mu\rho}\gamma^\nu + g^{\nu\rho}\gamma^\mu + i\epsilon^{\mu\nu\rho\alpha}\gamma_\alpha\gamma_5$$

$$\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma = (g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho})\mathbb{1}$$
$$+ i(-g^{\mu\nu}\sigma^{\rho\sigma} + g^{\mu\rho}\sigma^{\nu\sigma} - g^{\mu\sigma}\sigma^{\nu\rho}$$
$$- g^{\nu\rho}\sigma^{\mu\sigma} + g^{\nu\sigma}\sigma^{\mu\rho} - g^{\rho\sigma}\sigma^{\mu\nu}) - i\epsilon^{\mu\nu\rho\sigma}\gamma_5$$

$$\vdots$$

Proof: Multiply both sides of $(*)$ by Γ_B, take trace and apply orthonormality relations:

$$S = \sum_{A=1}^{16} a_A \Gamma^A$$

$$\mathrm{Tr}[S\Gamma_B] = \sum_{A=1}^{16} a_A \underbrace{\mathrm{Tr}[\Gamma^A\Gamma_B]}_{4\delta^A{}_B}$$

i.e. $$a_B = \frac{1}{4}\mathrm{Tr}[S\Gamma_B]$$ ∎