Conversion of one Weyl fermion to Majorana fermion

Weyl Lagrangian:

$$\mathcal{L} = \chi^\dagger i\bar\sigma^\mu \partial_\mu \chi - \tfrac{1}{2} m_M (\chi\chi + \chi^\dagger\chi^\dagger)$$

integrate half term by parts.

$$= \tfrac{1}{2}\chi^\dagger i\bar\sigma^\mu\partial_\mu\chi - \tfrac{1}{2}(\partial_\mu\chi^\dagger) i\bar\sigma^\mu \chi - \tfrac{1}{2} m_M(\chi\chi + \chi^\dagger\chi^\dagger)$$

$$= \tfrac{1}{2}\chi^\dagger i\bar\sigma^\mu\partial_\mu\chi + \tfrac{1}{2}\chi i\sigma^\mu\partial_\mu\chi^\dagger - \tfrac{1}{2} m_M(\chi\chi + \chi^\dagger\chi^\dagger)$$

Could use DHM stacking formula too

$$= \tfrac{1}{2}\begin{pmatrix}\chi_\gamma & \chi^{\dagger\dot\gamma}\end{pmatrix}\begin{pmatrix} m_M\epsilon^{\gamma\beta} & -i\epsilon^{\gamma\alpha}\sigma^\mu_{\alpha\dot\beta}\partial_\mu \\ -i\epsilon_{\dot\gamma\dot\beta}\bar\sigma^{\mu\dot\alpha\beta}\partial_\mu & m_M\epsilon_{\dot\gamma\dot\beta}\end{pmatrix}\begin{pmatrix}\chi_\beta \\ \chi^{\dagger\dot\beta}\end{pmatrix}$$

$$= \underbrace{\tfrac{1}{2}\begin{pmatrix}\chi_\gamma & \chi^{\dagger\dot\gamma}\end{pmatrix}\begin{pmatrix}-\epsilon^{\gamma\alpha} & \\ & -\epsilon_{\dot\gamma\dot\alpha}\end{pmatrix}}_{\text{recall rule for contracting } \epsilon \text{ with spinors.}}\begin{pmatrix}-m_M\delta_\alpha{}^\beta & i\sigma^\mu_{\alpha\dot\beta}\partial_\mu \\ i\bar\sigma^{\mu\dot\alpha\beta}\partial_\mu & -m_M\delta^{\dot\alpha}{}_{\dot\beta}\end{pmatrix}\begin{pmatrix}\chi_\beta \\ \chi^{\dagger\dot\beta}\end{pmatrix}$$

$$= \tfrac{1}{2}\begin{pmatrix}+\chi^\alpha & \chi^\dagger_{\dot\alpha}\end{pmatrix}\left[i\begin{pmatrix} & \sigma^\mu_{\alpha\dot\beta} \\ \bar\sigma^{\mu\dot\alpha\beta} & \end{pmatrix}\partial_\mu - m_M\begin{pmatrix}\delta_\alpha{}^\beta & \\ & \delta^{\dot\alpha}{}_{\dot\beta}\end{pmatrix}\right]\begin{pmatrix}\chi_\beta \\ \chi^{\dagger\dot\beta}\end{pmatrix}$$

$$= \tfrac{1}{2}\bar\psi_M(i\partial\!\!\!/ - m_M)\psi_M$$

$$\psi_M = \begin{pmatrix}\psi_L \\ \psi_L{}^c\end{pmatrix}$$

Majorana condition

$$(\psi_M)^c = \psi_M \quad\Rightarrow\quad \begin{pmatrix}\psi_L \\ (\psi_L)^c\end{pmatrix}^c = \begin{pmatrix}\psi_L \\ (\psi_L)^c\end{pmatrix}$$