Plane wave expansion

$$\chi_\alpha = \psi_L$$

$$\chi^{\dagger\dot{\alpha}} = (\psi_L)^c = (\psi^c)_R$$

$$\chi_\alpha = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega_p}} \sum_s \left[\hat{a}_{\vec{p},s}\, u_\alpha^{[s]}(\vec{p})\, e^{-ip\cdot x} + \hat{a}^\dagger_{\vec{p},s}\, v_\alpha^{[s]}(\vec{p})\, e^{ip\cdot x} \right]$$

$$\chi^{\dagger\dot{\alpha}} = \quad " \quad " \quad \left[\hat{a}_{\vec{p},s}\, v^{\dagger[s]\dot{\alpha}}(\vec{p})\, e^{-ip\cdot x} + \hat{a}^\dagger_{\vec{p},s}\, u^{\dagger[s]}(\vec{p})^{\dot{\alpha}}\, e^{ip\cdot x} \right]$$

so

$$\psi = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega_p}} \sum_s \left[\hat{a}_{\vec{p},s}\, u^{[s]}_{\text{Dirac}}(\vec{p})\, e^{-ip\cdot x} + \hat{a}^\dagger_{\vec{p},s}\, v^{[s]}_{\text{Dirac}}(\vec{p})\, e^{ip\cdot x} \right]$$

$$\bar{\psi} = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega_p}} \sum_s \left[\hat{a}^\dagger_{\vec{p},s}\, \bar{u}^{[s]}_{\text{Dirac}}(\vec{p})\, e^{ip\cdot x} + \hat{a}_{\vec{p},s}\, \bar{v}^{[s]}_{\text{Dirac}}(\vec{p})\, e^{-ip\cdot x} \right]$$

Anticommutation relations:
– Same as for Weyl spinors.

$$\{\hat{a}_{\vec{p},s}, \hat{a}^\dagger_{\vec{p}',s'}\} = (2\pi)^3 \delta^{(3)}(\vec{p}-\vec{p}')\, \delta_{ss'}$$

__Comment:__ Since $\overline{\psi\psi}$ and $\overline{\bar{\psi}\bar{\psi}}$ no longer vanish, many more contractions are allowed, complicating the Feynman rules.

e.g. Consider the Yukawa interaction: $\mathcal{L}_{int} = -\frac{1}{2} y\, \phi \bar{\psi}\psi$.

Feynman rules derived from __two__ contractions

$$\langle 0 |\, -\tfrac{1}{2} y \phi\, \bar{\psi}\psi\, | \psi\psi \rangle \quad \underline{\text{and}} \quad \langle 0 |\, -\tfrac{1}{2} y \phi\, \psi\psi\, | \psi\psi \rangle$$