Hiren H. Patel
hhpatel.net

Parity of Majorana Fermions

For one-fermion state, require:

- Momentum to flip $\vec{p} \longrightarrow -\vec{p}$
- Spin to remain unchanged $\vec{S} \longrightarrow \vec{S}$

Implementation as unitary operator:

$$\hat{P}\, \hat{a}_{\vec{p},s}\, \hat{P}^{-1} = \eta_a\, \hat{a}_{-\vec{p},s}$$

Restriction on phase:

observables are of the form $\sim a^\dagger a$

$\hat{P}^2$ should return obs. to original values: $\Rightarrow \eta_a = \{\pm 1, \pm i\}$.

Parity transformation of Majorana spinor:

$$\hat{P}\, \hat{\psi}_M(t,\vec{x})\, \hat{P}^{-1} = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega_p}} \sum_s \left[(\eta_a\, \hat{a}_{-\vec{p},s})\, u^{[s]}(\vec{p})\, e^{-ip\cdot x} + (\eta_a^*\, \hat{a}^\dagger_{-\vec{p},s})\, v^{[s]}(\vec{p})\, e^{ip\cdot x} \right]$$

ch. var: $\vec{p} \longrightarrow -\vec{p}$

use: $u(-\vec{p}) = \gamma^0 u(\vec{p})$ & $v(-\vec{p}) = -\gamma^0 v(-\vec{p})$

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega_p}} \sum_s \Big[\eta_a \gamma^0 \hat{a}_{\vec{p},s}\, u(\vec{p})\, e^{-ip\cdot\binom{t}{-\vec{x}}} + (-\eta_a^* \gamma^0)\, \hat{a}_{\vec{p},s}\, v(\vec{p})\, e^{ip\cdot\binom{t}{-\vec{x}}}$$

Can get good transformation property if $\underbrace{\eta_a = -\eta_a^*}_{\Downarrow} := \eta_P$

$$\hat{P}\, \hat{\psi}_M(t,\vec{x})\, \hat{P}^{-1} = \eta_P^*\, \gamma^0\, \hat{\psi}_M(t,-\vec{x}) \qquad \eta_P = \pm i \text{ only.}$$

Transformation rule of one and two particle states:

$$|1_{\vec{p},s}\rangle \xrightarrow{P} \eta_P^*\, |1_{-\vec{p},s}\rangle$$

$$|1_{\vec{p},s}, 1_{\vec{p}',s'}\rangle \xrightarrow{P} (\eta_P^*)^2\, |1_{-\vec{p},s}, 1_{-\vec{p}',s'}\rangle$$

$$= -\,|1_{-\vec{p},s}, 1_{-\vec{p}',s'}\rangle \qquad (!)$$

Two Majorana fermions have negative intrinsic parity.
(consistent with Weyl fermions)