<u>Atypical completeness relations</u>

(u, v cross relations)

→ To chiral basis:

$$\sum_{s=\pm 1/2} u^{[s]}_{\text{Dirac}}(\vec{p})\; v^{[s]}_{\text{Dirac}}(\vec{p})^T$$

$$= \sum_s \begin{pmatrix} u_\alpha^{[s]}(\vec{p}) \\ v^{\dagger\dot{\alpha}[s]}(\vec{p}) \end{pmatrix} \otimes \begin{pmatrix} v_\beta^{[s]}(\vec{p}) & u^{\dagger\dot{\beta}[s]}(\vec{p}) \end{pmatrix}$$

$$= \sum_s \begin{pmatrix} u_\alpha v^\gamma \epsilon_{\beta\gamma} & u_\alpha u^\dagger_{\dot{\gamma}} \epsilon^{\dot{\beta}\dot{\gamma}} \\ v^{\dagger\dot{\alpha}} v^\gamma \epsilon_{\beta\gamma} & v^{\dagger\dot{\alpha}} u^\dagger_{\dot{\gamma}} \epsilon^{\dot{\beta}\dot{\gamma}} \end{pmatrix}$$

$$= \begin{pmatrix} m\,\delta_\alpha{}^\gamma \epsilon_{\beta\gamma} & p\cdot\sigma_{\alpha\dot{\gamma}} \epsilon^{\dot{\beta}\dot{\gamma}} \\ p\cdot\bar{\sigma}^{\dot{\alpha}\gamma} \epsilon_{\beta\gamma} & m\,\delta^{\dot{\alpha}}{}_{\dot{\gamma}} \epsilon^{\dot{\beta}\dot{\gamma}} \end{pmatrix}$$

$$= \begin{pmatrix} m\,\delta_\alpha{}^\gamma & p\cdot\sigma_{\alpha\dot{\gamma}} \\ p\cdot\bar{\sigma}^{\dot{\alpha}\gamma} & m\,\delta^{\dot{\alpha}}{}_{\dot{\gamma}} \end{pmatrix} \begin{pmatrix} \epsilon_{\beta\gamma} & \\ \ominus & \epsilon^{\dot{\beta}\dot{\gamma}} \end{pmatrix}$$

$$= \ominus\; (\not{p} + m)_{\alpha\gamma} \underbrace{\begin{pmatrix} \epsilon_{\gamma\beta} & \\ & \epsilon^{\dot{\beta}\dot{\gamma}} \end{pmatrix}}_{\begin{pmatrix} 0 & -1 & & \\ +1 & 0 & & \\ & & 0 & +1 \\ & & -1 & 0 \end{pmatrix} \;=\; -i\gamma^2\gamma^0}$$

$$= (\not{p} + m)\, \underbrace{i\gamma^2\gamma^0}_{C}$$

Similarly,

$$\sum_{s=\pm 1/2} \bar{u}^{[s]}_{\text{Dirac}}(\vec{p})^T\; \bar{v}^{[s]}_{\text{Dirac}}(\vec{p}) = \underbrace{i\gamma^2\gamma^0}_{C} (\not{p} + m)$$