<u>Full fermion propagator</u> (incl. $\cancel{P}$ & $\cancel{CP}$ terms)

Lagrangian: $\mathcal{L}_0 = \bar{\psi}\,(i\cancel{\partial} - m_R - i m_I \gamma_5)\,\psi$.

Bare propagator: $$[\tilde{S}_0(\cancel{p})]^{-1} = -i\,(\cancel{p} - m_R - i m_I \gamma_5) \qquad [1]$$

<u>Full propagator:</u>

First define self energy function (1PI)

$$-i\Sigma(\cancel{p}) = \text{—(1PI)—} := -i\,(\cancel{p}A + m_R' B + \cancel{p}\gamma_5 C + i m_I' \gamma_5 E) \qquad [2]$$

If bare masses vanish, can choose another mass here.

— conventional choice: $m_R' = m_I' = m_R$

In terms of $-i\Sigma(\cancel{p})$, the full propagator is:

$$\tilde{S}_{\text{Full}}(\cancel{p}) = \text{——}$$

$$= \text{———} + \text{—(1PI)—} + \text{—(1PI)(1PI)—} \qquad \text{Dyson's expansion}$$

$$= \tilde{S}_0 + \tilde{S}_0[-i\Sigma]\tilde{S}_0 + \tilde{S}_0[-i\Sigma]\tilde{S}_0[-i\Sigma]\tilde{S}_0 + \ldots$$

$$= \tilde{S}_0 \sum_{n=0}^{\infty} (-i\Sigma\,\tilde{S}_0) \qquad \text{Geometric series}$$

$$= \tilde{S}_0\,(1 + i\Sigma\,\tilde{S}_0)^{-1}$$

Take matrix inv. of both sides:

$$[\tilde{S}_{\text{Full}}(\cancel{p})]^{-1} = (1 + i\Sigma\,\tilde{S}_0)\,\tilde{S}_0^{-1}$$

$$= \tilde{S}_0^{-1} + i\Sigma \qquad [3]$$

Insert parametric forms [1] & [2]:

$$[\tilde{S}_{\text{Full}}(\cancel{p})]^{-1} = -i(\cancel{p} - m_R - i m_I\gamma_5) + i(\cancel{p}A + m_R' B + \cancel{p}\gamma_5 C + i m_I' \gamma_5 E)$$

$$= -i\Big((1-A)\cancel{p} - (m_R + B m_R') - C\cancel{p}\gamma_5 - i(m_I + E m_I')\gamma_5\Big)$$

Invert this to obtain full propagator:

Formally, $$\tilde{S}_{\text{Full}}(\cancel{p}) = \frac{i}{\cancel{p} - m_R - i m_I\gamma_5 - \Sigma(\cancel{p})}$$

Matrix inversion:

Abbreviate
$a = (1-A)$
$b = (m_R + B m_R')$
$c = -C$
$e = -i(m_I + E m_I')$

So that $\left[\tilde{S}_{Full}(\not{p})\right]^{-1} = -i(a\not{p} - b + c\not{p}\gamma_5 + e\gamma_5)$

Ansatz: $\tilde{S}_{Full}(\not{p}) = i(a'\not{p} - b' + c'\not{p}\gamma_5 + e'\gamma_5)$ a', b', c', e' to be determined.

Then

$$\tilde{S}_{Full}^{-1} \cdot \tilde{S}_{Full} = \mathbb{1} = (a\not{p} - b + c\not{p}\gamma_5 + e\gamma_5) \cdot (a'\not{p} - b' + c'\not{p}\gamma_5 + e'\gamma_5)$$

$$= (aa'p^2 + bb' - cc'p^2 + zz')\,\mathbb{1}$$

$$+ (-(ab' + a'b) + (ce' + c'e))\not{p}$$

$$+ ((ac' - a'c)p^2 - (be' + b'e))\gamma_5$$

$$+ ((ae' - a'e) + (bc' + b'c))\not{p}\gamma_5$$

$\therefore$ Require 1^{st} line $= 1$
and $2^{nd} = 3^{rd} = 4^{th}$ lines $= 0$

Solution:

Common denominator to $a'b'c'e'$: $[(1-A)^2 - C^2]p^2 - (m_R + B m_R')^2 - (m_I + E m_I')^2$

$a' = 1 - A$
$b' = -1 - B$
$c' = -iC$
$e' = -i(1+E)$

$$\tilde{S}_{Full}(\not{p}) = \frac{i\left[(1-A)\not{p} + (m_R + B m_R') - C\not{p}\gamma_5 - i(m_I + E m_I')\gamma_5\right]}{\left[(1-A(p^2))^2 - C(p^2)^2\right]p^2 - (m_R + B(p^2) m_R')^2 - (m_I + E(p^2) m_I')^2}$$

Usually, the axial phase of bare fermion field chosen so that $m_I = 0$. $\quad \mathcal{L} = \bar\psi(i\not\partial - m)\psi$

$\boxed{\text{If } m_R \neq 0,}$ the convention is $m_R' = m_I' = m_R := m$

$$\tilde{S}_{\text{Full}}(\not p) = \frac{i[(1-A)\not p + (1+B)m - C\not p\gamma_5 - iEm\gamma_5]}{[(1-A)^2 - C^2]p^2 - [(1+B)^2 + E^2]m^2}$$

<u>One loop pole mass:</u>

$$\text{denom}\Big|_{p^2 = m_{pole}^2} = 0$$

$$[(1-A(p^2))^2 - C(p^2)^2]p^2 - [(1+B(p^2))^2 + E(p^2)^2]m^2\Big|_{p^2 = m_{pole}^2} = 0.$$

View m_{pole} as analytic function of $\hbar$
(m_{pole}^2 is for bosons)

$$m_{pole} = m_0 + \hbar m_1 + \hbar^2 m_2 + \ldots$$
$$A(p^2) = \hbar A_1 + \hbar^2 A_2 + \ldots$$
$$B(p^2) = \hbar B_1 + \ldots$$
$$C(p^2) = \ldots$$
$$E(p^2) = \ldots$$

$$(1 - 2A + \underbrace{A^2 - C^2}_{\text{small}})(m_0^2 + 2\hbar m_0 m_1 + \ldots) - [1 + 2B + \underbrace{B^2 + E^2}_{\text{small}}]m^2 = 0$$

$\mathcal{O}(\hbar^0)$: $\quad m_0^2 = m^2$

$\mathcal{O}(\hbar^1)$: $\quad 2\hbar m_0 m_1 - 2\hbar A_1 m_0^2 - 2\hbar B_1 m^2 = 0$

$$\underline{m_1 = m_0[A_1(m_0^2) + B_1(m_0^2)]}$$

$\boxed{\text{If } m_R = 0}$, the convention is $m_R' = m_I' := M$ ← unrelated mass scale associated with chiral symmetry breaking

$$\tilde{S}_{\text{Full}}(\not p) = \frac{i[(1-A)\not p + BM - C\not p\gamma_5 - iEM\gamma_5]}{[(1-A)^2 - C^2]p^2 - [B^2 + E^2]M^2}.$$

<u>One loop pole mass:</u> $\quad m_{pole} = m_0 + \hbar m_1 + \ldots$

$$m_0 = 0$$
$$m_1 = M\sqrt{B_1^2(0) + E_1^2(0)}$$