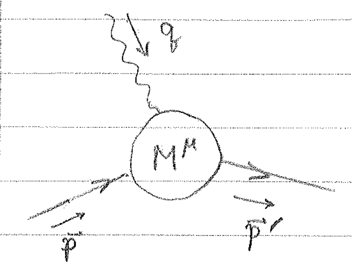
Parametrizations for vector vertex matrix



Kinematics: $q^\mu = (p'-p)^\mu$ $q^2 < 0$ (space-like)

External fermions on-shell:

$$p^2 = m_1^2$$
$$p'^2 = m_2^2$$
$$p\cdot p' = \tfrac{1}{2}(-q^2 + m_1^2 + m_2^2)$$
$$p\cdot q = \tfrac{1}{2}(-q^2 - m_1^2 + m_2^2)$$
$$p'\cdot q = \tfrac{1}{2}(q^2 - m_1^2 + m_2^2)$$

Define:

$$P^\mu = \tfrac{1}{2}(p+p')^\mu$$
$$P^2 = \tfrac{1}{4}(2m_1^2 + 2m_2^2 - q^2)$$
$$P\cdot q = 0$$

① Vector/axial-vector parametrization: [Package-X convention]

$$\bar{u}(\vec{p}\,')\, M^\mu\, u(\vec{p}) = \bar{u}(\vec{p}\,')\Big[\Big(\gamma^\mu - \frac{\not{q}\, q^\mu}{q^2}\Big) F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{m_1+m_2} F_2(q^2) + \frac{2q^\mu}{m_1+m_2} F_3(q^2)$$
$$+ \Big(\gamma^\mu - \frac{\not{q}\, q^\mu}{q^2}\Big)\gamma_5\, G_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu\gamma_5}{m_1+m_2} G_2(q^2) + \frac{2q^\mu}{m_1+m_2}\gamma_5\, G_3(q^2)\Big] u(\vec{p})$$

In $m_1 = m_2 \equiv m$ limit, [apply $\bar{u}\not{q}u = 0$]

$$\longrightarrow \bar{u}(\vec{p}\,')\Big[\gamma^\mu F_1 + \frac{i\sigma^{\mu\nu}q_\nu}{2m} F_2 + \frac{q^\mu}{m} F_3$$
$$+ \Big(\gamma^\mu - \frac{\not{q}\, q^\mu}{q^2}\Big)\gamma_5\, G_1 + \frac{i\sigma^{\mu\nu}q_\nu\gamma_5}{2m} G_2 + \frac{q^\mu}{m}\gamma_5\, G_3\Big] u(\vec{p})$$

F_1 = Dirac form factor
F_2 = Pauli form factor
G_1 = Anapole form factor
G_2 = EDM form factor

F_3 & G_3 violate $q_\mu \bar{u} M^\mu u = 0$.
$\Rightarrow$ must vanish if current is conserved.

② Left/right chiral parametrization

$$\bar{u}(\vec{p}\,') M^\mu u(\vec{p}) = \bar{u}(\vec{p}\,')\Big[\left(\gamma^\mu - \frac{\not{q}\, q^\mu}{q^2}\right)\hat{P}_L\, A_L(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{m_1+m_2}\hat{P}_L\, B_L(q^2) + \frac{2q^\mu}{m_1+m_2}\hat{P}_L\, C_L(q^2)$$

$$+\left(\gamma^\mu - \frac{\not{q}\, q^\mu}{q^2}\right)\hat{P}_R\, A_R(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{m_1+m_2}\hat{P}_R\, B_R(q^2) + \frac{2q^\mu}{m_1+m_2}\hat{P}_R\, C_R(q^2)\Big] u(\vec{p}).$$

A_L, B_L, A_R, B_R are "dipole" form factors.

C_L & C_R violate $q_\mu \bar{u} M^\mu u = 0$ $\Rightarrow$ must vanish if current is conserved.

connection formulae:

(inverse relations)

$$A_L = F_1 - G_1 \qquad A_R = F_1 + G_1 \qquad F_1 = \tfrac{1}{2}(A_L + A_R) \qquad G_1 = \tfrac{1}{2}(-A_L + A_R)$$

$$B_L = F_2 - G_2 \qquad B_R = F_2 + G_2 \qquad F_2 = \tfrac{1}{2}(B_L + B_R) \qquad G_2 = \tfrac{1}{2}(-B_L + B_R)$$

$$C_L = F_3 - G_3 \qquad C_R = F_3 + G_3 \qquad F_3 = \tfrac{1}{2}(C_L + C_R) \qquad G_3 = \tfrac{1}{2}(-C_L + C_R)$$

Secondary form factors

Sachs Electric & Magnetic:

$$\bar{u}\left[G_E(q^2)\,\frac{2m\,(p'+p)^\mu}{4m^2-q^2} + G_M(q^2)\left(\frac{-q^2(p'+p)^\mu}{2m(4m^2-q^2)} + \frac{1}{2m}\, i\sigma^{\mu\nu} q_\nu\right)\right] u$$

in terms of F_1 & F_2

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m^2} F_2(q^2)$$

$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$

$\Longrightarrow$

inverse relations:

$$F_1(q^2) = \frac{1}{4m^2-q^2}\left(4m^2 G_E - q^2 G_M\right)$$

$$F_2(q^2) = \frac{4m^2}{4m^2-q^2}\left(-G_E + G_M\right)$$

Anapole moment

$$\bar{u}\left[F_A(q^2)\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\gamma_\nu\gamma_5\right] u = \bar{u}\left[F_A(q^2)\left(\gamma^\mu - \frac{2mq^\mu}{q^2}\right)\gamma_5\right] u$$

In terms of G_1 & G_3:

$$F_A = \frac{1}{2}\left[G_1(q^2) - \frac{q^2}{2m^2} G_3(q^2)\right]$$