Gordon identity — momentum space

Relates convective transition currents to Dirac and Pauli transition currents:

Let w stand for u or v spinor $\quad \not{p}\, w_p = \varepsilon m w_p$

frequency: $\varepsilon = +1 \quad \omega = -1$ $\quad \bar{w}_{p'} \not{p}' = \bar{w}_{p'} \varepsilon' m'$

$$\bar{w}_{p'} \sigma^{\mu\nu} (\varepsilon' p' - \varepsilon p)_\nu \gamma_5 w_p$$

$$= \bar{w}_{p'} \frac{i}{2} (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu)(\varepsilon' p' - \varepsilon p)_\nu \gamma_5 w_p$$

$$= \bar{w}_{p'} \frac{i}{2} \left[\gamma^\mu (\varepsilon' \not{p}' - \varepsilon \not{p}) - (\varepsilon' \not{p}' - \varepsilon \not{p}) \gamma^\mu \right] \gamma_5 w_p$$

$$= \bar{w}_{p'} \frac{i}{2} \left[\gamma^\mu \varepsilon' \not{p}' - \gamma^\mu \varepsilon \not{p} - \varepsilon' \not{p}' \gamma^\mu + \varepsilon \not{p} \gamma^\mu \right] \gamma_5 w_p$$

$$= \bar{w}_{p'} \frac{i}{2} \left[-\varepsilon' \not{p}' \gamma^\mu + 2\varepsilon' p'^\mu \overset{(+)}{} m\gamma^\mu - m'\gamma^\mu - \varepsilon \gamma^\mu \not{p} + 2\varepsilon p^\mu \right] \gamma_5 w_p$$

$$= \bar{w}_{p'} \frac{i}{2} \left[-m'\gamma^\mu + 2\varepsilon' p'^\mu \overset{(+)}{} m\gamma^\mu - m'\gamma^\mu \overset{(+)}{} m\gamma^\mu + 2\varepsilon p^\mu \right] \gamma_5 w_p$$

$$= \bar{w}_{p'} \frac{i}{2} \left[-2(m' \overset{(+)}{-} m)\gamma^\mu + 2(\varepsilon' p' + \varepsilon p)^\mu \right] \gamma_5 w_p$$

$\therefore$

$$\bar{w}_{p'} i\sigma^{\mu\nu} (\varepsilon' p' - \varepsilon p)_\nu \gamma_5 w_p = \bar{w}_{p'} \left[(m' \overset{(+)}{-} m)\gamma^\mu - \underbrace{(\varepsilon' p' + \varepsilon p)^\mu}_{\text{solve}} \right] \gamma_5 w_p$$

$$\boxed{\bar{w}_{p'} (\gamma_5) w_p \overbrace{(\varepsilon' p' + \varepsilon p)^\mu}^{P^\mu} = (m' \overset{(+)}{-} m)\, \bar{w}_{p'} \gamma^\mu \gamma_5 w_p - \bar{w}_{p'}\, i\sigma^{\mu\nu} \overbrace{(\varepsilon' p' - \varepsilon p)_\nu}^{q} \gamma_5 w_p}$$

convective current — Dirac current — Pauli current

also written:

$$\bar{w}_{p'} \gamma^\mu (\gamma_5) w_p = \bar{w}_{p'} \left[\frac{(\varepsilon' p' + \varepsilon p)^\mu}{m' \overset{(+)}{-} m} + \frac{i\sigma^{\mu\nu} (\varepsilon' p' - \varepsilon p)_\nu}{m' \overset{(+)}{-} m} \right] (\gamma_5) w_p$$