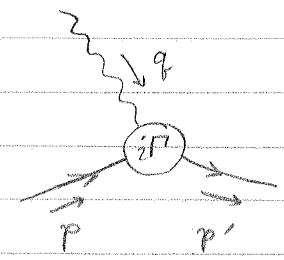
Czarnecki and Krause (1996)
Dipole moments.

Projectors for vector/axial-vector form factors



Kinematics:

$$(p' - p)^\mu = q^\mu$$

External fermion on-shell:

$$p^2 = p'^2 = m^2$$

$$p \cdot p' = m^2 - \tfrac{1}{2} q^2$$

$$p \cdot q = -\tfrac{1}{2} q^2$$

$$p' \cdot q = \tfrac{1}{2} q^2$$

Define:

$$P^\mu = \tfrac{1}{2}(p + p')$$

$$P^2 = m^2 - \tfrac{1}{4} q^2$$

$$P \cdot q = 0$$

Most general matrix element (used Gordon id.)

$$\boxed{\begin{aligned} i\mathcal{M}^\mu = \underbrace{-ie}_{\text{normalization of current.}} \bar{u}(\vec{p}') \Big[& F_1(q^2)\gamma^\mu + \frac{i}{2m} F_2(q^2)\sigma^{\mu\nu} q_\nu + \frac{1}{m} F_3(q^2) q^\mu \\ & + \Big(G_1(q^2)\gamma^\mu + \frac{i}{2m} G_2(q^2)\sigma^{\mu\nu} q_\nu + \frac{1}{m} G_3(q^2) q^\mu \Big)\gamma_5 \Big] u(\vec{p}) \end{aligned}}$$

Define, $\Gamma^\mu = [\ldots]$ (u & $\bar{u}$ spinors removed)

If vector current conserved $F_3(q^2) = 0$

~~If axial-vector current conserved, $G_3(q^2) = 0$.~~

Projector:

$$\mathcal{F}_\mu = (\not{p} + m)\Big[f_1 \gamma_\mu + \frac{1}{m} f_2 P_\mu + \frac{1}{m} f_3 q_\mu + \Big(g_1 \gamma_\mu + \frac{1}{m} g_2 P_\mu + \frac{1}{m} g_3 q_\mu \Big)\gamma_5 \Big](\not{p}' + m)$$

where $f_1, f_2, f_3, g_1, g_2, g_3$ are to be determined by computing $\mathrm{Tr}[\mathcal{F}_\mu (\Gamma^\mu)]$.

Note: the $(\not{p}+m)$ and $(\not{p}'+m)$ at ends will act as u & $\bar{u}$ spinors enforcing $\not{p} u = m u$.

$$
\begin{aligned}
\mathrm{Tr}[\mathcal{F}_\mu \Gamma^\mu] = \; & \Big[\big(8m^2 + 2q^2(d-2)\big) f_1 + (8m^2 - 2q^2) f_2 \Big] F_1(q^2) \\
& + \Big[2(d-1) q^2 f_1 + q^2\Big(2 - \frac{q^2}{2m^2}\Big) f_2 \Big] F_2(q^2) \\
& + \Big[2q^2 \Big(4 - \frac{q^2}{m^2}\Big) f_3 \Big] F_3(q^2) \\
& + \Big[\big(-8m^2(d-1) + 2q^2(d-2)\big) g_1 + 4q^2 g_3 \Big] G_1(q^2) \\
& + \Big[q^2 \Big(-2 + \frac{q^2}{2m^2}\Big) g_2 \Big] G_2(q^2) \\
& + \Big[-4q^2 g_1 + \frac{2q^2}{m^2} g_3 \Big] G_3(q^2)
\end{aligned}
$$

To get the relevant projections,
set all coefficients equal to zero, except for the one
in which we are interested. $\rightarrow$ solve.

Dirac: $F_1(q^2)$: $f_1 = \dfrac{-1}{2(4m^2-q^2)(d-2)}$, $f_2 = \dfrac{2m^2}{(4m^2-q^2)^2} \dfrac{(d-1)}{(d-2)}$

Pauli: $F_2(q^2)$: $f_1 = \dfrac{2m^2}{(4m^2-q^2)q^2(d-2)}$, $f_2 = -\dfrac{2m^2\big(4m^2+q^2(d-2)\big)}{(4m^2-q^2)^2 q^2 (d-2)}$

$F_3(q^2)$: $f_3 = \dfrac{+m^2}{2q^2(4m^2-q^2)}$

$G_1(q^2)$: $g_1 = \dfrac{-1}{2(d-2)} \dfrac{1}{4m^2-q^2}$, $g_3 = \dfrac{-1}{d-2} \dfrac{m^2}{q^2(4m^2-q^2)}$

EDM $G_2(q^2)$: $g_2 = \dfrac{-2m^2}{q^2(4m^2-q^2)}$

$G_3(q^2)$: $g_1 = \dfrac{1}{d-2} \dfrac{m^2}{q^2(4m^2-q^2)}$ $g_3 = \dfrac{m^2}{2q^4} + \dfrac{2}{d-2} \dfrac{m^4}{q^4(4m^2-q^2)}$

Example:

Now use the projector for G_2 to extract EDM.

$$\mathcal{F}_\mu^{[G_2]} = (\not{p}+m)\,\frac{1}{m}\,\frac{-2m^2}{q^2(4m^2-q^2)}\,P_\mu\,\gamma_5\,(\not{p}'+m)$$

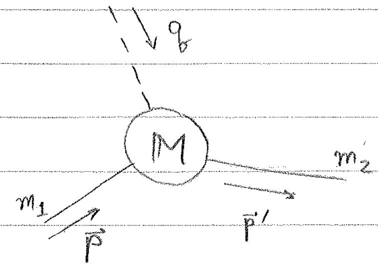
$$= \frac{-2m}{q^2(4m^2-q^2)}\,P_\mu\,(\not{p}+m)\,\gamma_5\,(\not{p}'+m)$$

$$= \frac{-m}{q^2(4m^2-q^2)}\,(p+p')_\mu\,(\not{p}+m)\,\gamma_5\,(\not{p}'+m)$$

Then,

$$G_2(q^2) = \mathrm{Tr}\left[\frac{-m}{q^2(4m^2-q^2)}\,\Gamma^\mu(q^2)\right]$$

Parametrization/projection for scalar form factor



Parametrization:

$$\bar{u}(\vec{p}')\, \mathbb{M}\, u(\vec{p}) = \bar{u}(\vec{p}') \left[\mathbb{1}\, G_s(q^2) + i\gamma_5\, G_P(q^2) \right] u(\vec{p})$$

Projector:

$$\mathcal{F}^{[\;]} = (\not{p} + m_1) \left[\mathbb{1}\, g_s + \gamma_5\, g_P \right] (\not{p}' + m_2)$$

Multiply, and take trace:

$$\mathrm{Tr}\left[\mathbb{M}\, \mathcal{F} \right] = G_s\, g_s\, 2\left(-q^2 + (m_1+m_2)^2 \right) + G_P\, g_P\, 2i\left(-q^2 + (m_1-m_2)^2 \right)$$

To project G_s, $\quad g_s = \dfrac{1}{2(-q^2+(m_1+m_2)^2)}$ & $g_P = 0$

" $\quad G_P$, $\quad g_s = 0$ & $g_P = \dfrac{1}{2i(-q^2+(m_1-m_2)^2)}$

$\therefore$

$$\mathcal{F}^{[S]} = \frac{-(\not{p}+m_1)(\not{p}'+m_2)}{2\left[q^2-(m_1+m_2)^2\right]} \quad \text{and} \quad \mathcal{F}^{[P]} = \frac{(\not{p}+m_1)\gamma_5(\not{p}'+m_2)}{2i\left[-q^2+(m_1-m_2)^2\right]}$$