Electromagnetic dipole moments

$$\mathcal{L}_{int} = -\frac{a}{2}\frac{e\hbar}{2m} F_{\mu\nu}\,\bar{\psi}\,\sigma^{\mu\nu}\psi \;-\; \frac{i\hbar d}{2} F_{\mu\nu}\,\bar{\psi}\,\sigma^{\mu\nu}\gamma_5\psi$$

$$\Big\downarrow \text{ one particle Hamiltonian}$$

$$\hat{H}_{int} = +\frac{a}{2}\underbrace{\frac{e\hbar}{2m}}_{\mu_B} F_{\mu\nu}\,\gamma^0\sigma^{\mu\nu} \;+\; \frac{i\hbar d}{2} F_{\mu\nu}\,\gamma^0\sigma^{\mu\nu}\gamma_5$$

Salpeter Hamiltonian

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ & 0 & -B_z & B_y \\ & & 0 & -B_x \\ & & & 0 \end{pmatrix} \qquad \sigma^{\mu\nu} = \begin{pmatrix} 0 & \begin{pmatrix} & i\sigma^1 \\ i\sigma^1 & \end{pmatrix} & \begin{pmatrix} & i\sigma^2 \\ i\sigma^2 & \end{pmatrix} & \begin{pmatrix} & i\sigma^3 \\ i\sigma^3 & \end{pmatrix} \\ & 0 & \begin{pmatrix} \sigma^3 & \\ & \sigma^3 \end{pmatrix} & \begin{pmatrix} -\sigma^2 & \\ & -\sigma^2 \end{pmatrix} \\ & & 0 & \begin{pmatrix} \sigma^1 & \\ & \sigma^1 \end{pmatrix} \\ & & & 0 \end{pmatrix} \qquad \gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

[Standard basis].

$$F_{\mu\nu}\sigma^{\mu\nu} = \begin{pmatrix} -2\vec{B}\cdot\vec{\sigma} & 2i\vec{E}\cdot\vec{\sigma} \\ 2i\vec{E}\cdot\vec{\sigma} & -2\vec{B}\cdot\vec{\sigma} \end{pmatrix} = -2\vec{B}\cdot\vec{\Sigma} + 2i\vec{E}\cdot\vec{\alpha}$$

$$F_{\mu\nu}\sigma^{\mu\nu}\gamma_5 = \begin{pmatrix} 2i\vec{E}\cdot\vec{\sigma} & -2\vec{B}\cdot\vec{\sigma} \\ -2\vec{B}\cdot\vec{\sigma} & 2i\vec{E}\cdot\vec{\sigma} \end{pmatrix} = 2i\vec{E}\cdot\vec{\Sigma} - 2\vec{B}\cdot\vec{\alpha}$$

Therefore, the one-particle Dirac Hamiltonian is:

$$\hat{H}_{int} = \frac{a}{2}\mu_B\,\gamma^0\left(-2\vec{B}\cdot\vec{\Sigma} + 2i\vec{E}\cdot\vec{\alpha}\right) + \frac{i\hbar d}{2}\gamma^0\left(2i\vec{E}\cdot\vec{\Sigma} - 2\vec{B}\cdot\vec{\alpha}\right)$$

$$= -a\mu_B\left(\vec{B}\cdot(\gamma^0\vec{\Sigma}) - i\vec{E}\cdot\vec{\gamma}\right) - \hbar d\left(\vec{E}\cdot(\gamma^0\vec{\Sigma}) + i\vec{B}\cdot\vec{\gamma}\right)$$

Connection to static electric and magnetic dipole moments

Chain of matching:

Feynman diagram calculation $\longrightarrow$ Effective Salpeter Lagrangian $\longrightarrow$ Non-relativistic Pauli Hamiltonian

Start from relativistic effective action:

$$\Gamma_{eff} = \int d^4x \, \mathcal{L}_{eff}$$

optional

$$\mathcal{L}_{eff} = \frac{-a}{2} \frac{Qe}{2m} F_{\mu\nu} \bar{\psi} \sigma^{\mu\nu} \psi - \frac{id}{2} F_{\mu\nu} \bar{\psi} \sigma^{\mu\nu} \gamma_5 \psi$$

Hamiltonian:

$$H_{eff} = -L_{eff} \qquad \text{(no derivative interactions)}$$

$$= \int d^3x \left[\frac{+a}{2} \frac{Qe}{2m} F_{\mu\nu} \underset{a^\dagger + b}{\underset{\wr}{\psi^\dagger}} \gamma^0 \sigma^{\mu\nu} \underset{a + b^\dagger}{\underset{\wr}{\psi}} + \frac{id}{2} F_{\mu\nu} \psi^\dagger \gamma^0 \sigma^{\mu\nu} \gamma_5 \psi \right]$$

one particle Hamiltonian:
(not antiparticle)

$$H_{1\text{-part}}^{(\vec{p}'s';\, ps)} = \langle \vec{p}'s' | \hat{H}_{eff} | \vec{p}s \rangle$$

$$= \frac{a}{2} \frac{Qe}{2m} F_{\mu\nu} \gamma^0 \sigma^{\mu\nu} + \frac{id}{2} F_{\mu\nu} \gamma^0 \sigma^{\mu\nu} \quad \longleftarrow \text{interaction Hamiltonian to be added to Dirac Hamiltonian}$$

Then follow QM notes to obtain (adding $\hat{H}_0$) Pauli's equation

$$i\hbar \frac{\partial}{\partial t} \phi = \left[\frac{\hat{p}^2}{2m} - \underbrace{\frac{Qe}{2m}(2+2a)}_{\vec{\mu}} \hat{\vec{S}} \cdot \vec{B} - \underbrace{2 d_E}_{\vec{d}} \hat{\vec{S}} \cdot \vec{E} - \frac{e}{m} \vec{A} \cdot \hat{\vec{p}} + \frac{e^2 \vec{A}^2}{2m} \right] \phi$$

ϕ: Pauli 2-component wavefunction

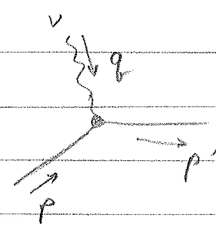
Next, match Feynman diagram calculation to effective action.

Feynman rule for:

$$\mathcal{L}_{eff} = c\, F_{\mu\nu}\, \bar{\psi}\, \sigma^{\mu\nu} \Gamma \psi \qquad \text{MDM: } c = \frac{-a}{2}\frac{Qe}{2m} \quad \Gamma = \mathbb{1} \qquad \text{EDM: } c = -\frac{id}{2} \quad \Gamma = \gamma_5$$

$$= c(\partial_\mu A_\nu - \partial_\nu A_\mu)\, \underset{\substack{\uparrow \\ \text{already antisymm.}}}{\bar{\psi}\sigma^{\mu\nu}} \Gamma \psi$$

$$= 2c\, \partial_\mu A_\nu\, \bar{\psi}\sigma^{\mu\nu}\Gamma\psi$$



Feynman rule

$$= (i)\, 2c \langle \psi_{\vec{p}'} | \partial_\mu A_\alpha\, \bar{\psi}\sigma^{\mu\alpha}\Gamma\psi | \psi_{\vec{p}}, A_{\vec{q}\nu} \rangle$$

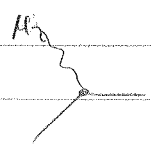
$$= i\,2c\,(-iq)_\mu\, g_{\alpha\nu}\, \sigma^{\mu\alpha}\Gamma$$

$$= 2c\, q_\mu (\sigma^{\mu\nu})\Gamma \qquad \text{ex. } \mu \Leftrightarrow \nu \Rightarrow \ominus$$

$$= -2c\, \sigma^{\nu\mu} q_\mu\, \Gamma$$

Therefore, Feynman rule for Salpeter effective Lagrangian

$$\mathcal{L}_{eff} = \frac{-a}{2}\frac{Qe\hbar}{2m} F_{\mu\nu}\bar{\psi}\sigma^{\mu\nu}\psi - \frac{i\hbar d}{2} F_{\mu\nu}\bar{\psi}\sigma^{\mu\nu}\gamma_5\psi$$



$$= a\frac{Qe}{2m}\sigma^{\mu\nu}q_\nu + id\,\sigma^{\mu\nu}q_\nu\gamma_5$$

Match to parametrization:

$$= -ie\left(\frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(0) + \frac{i\sigma^{\mu\nu}q_\nu\gamma_5}{2m}G_2(0)\right)$$

$$F_2(0) = aQ \quad \Rightarrow \quad a = F_2(0)/Q \quad \Rightarrow \quad \boxed{\vec{\mu} = \left(\frac{Qe}{m} + \frac{e}{m}F_2(0)\right)\vec{S}}$$

$$G_2(0) = 2m \qquad d = -\frac{ie}{2m}G_2(0) \Rightarrow \boxed{\vec{d} = \frac{-ie}{m}G_2(0)\,\vec{S}}$$