Levi-Civita tensor in 4D (Peskin-Schröder convention)

$$\epsilon^{\mu\nu\rho\sigma} = \begin{cases} +1 & \text{if } \underline{\text{even}} \text{ perm of } 0,1,2,3 \\ -1 & \text{if } \underline{\text{odd}} \text{ perm of } 0,1,2,3 \end{cases}$$

example

$\epsilon^{0123} = +1$

$\epsilon^{1230} = -1$

$$\epsilon_{\mu\nu\rho\sigma} = g_{\mu\mu'} g_{\nu\nu'} g_{\rho\rho'} g_{\sigma\sigma'} \epsilon^{\mu'\nu'\rho'\sigma'} = ?$$

Substitute $\mu=0$, $\nu=1$, $\rho=2$, $\sigma=3$.

$$\epsilon_{0123} = g_{0\mu'} g_{1\nu'} g_{2\rho'} g_{3\sigma'} \epsilon^{\mu'\nu'\rho'\sigma'}$$

only $\mu'=0$, $\nu'=1$, $\rho'=2$, $\sigma'=3$ terms survive

$$= (1)\underbrace{(-1)(-1)(-1)}\epsilon^{0123}$$

$$= -\epsilon^{0123}$$

$$= -1$$

Therefore,

$$\epsilon_{\mu\nu\rho\sigma} = \begin{cases} -1 & \text{if } \underline{\text{even}} \text{ perm of } 0,1,2,3 \\ +1 & \text{if } \underline{\text{odd}} \text{ perm of } 0,1,2,3 \end{cases}$$

<u>opposite</u> to that of ϵ^{0123}

Contraction of Minkowski Levi-Cinta tensor

$$\epsilon_{\mu\nu\rho\sigma}\,\epsilon^{\mu'\nu'\rho'\sigma'} = - \begin{vmatrix} \delta_\mu{}^{\mu'} & \delta_\mu{}^{\nu'} & \delta_\mu{}^{\rho'} & \delta_\mu{}^{\sigma'} \\ \delta_\nu{}^{\mu'} & \cdot & & \vdots \\ \delta_\rho{}^{\mu'} & & \cdot & \vdots \\ \delta_\sigma{}^{\mu'} & \cdot & \cdot & \delta_\sigma{}^{\sigma'} \end{vmatrix} = -\delta^{\mu'\nu'\rho'\sigma'}_{\mu\nu\rho\sigma}$$

Minkowski metric

[Generalized Kronecker Delta]

$$\epsilon_{\mu\nu\rho\sigma}\,\epsilon^{\mu'\nu'\rho'\sigma} = - \begin{vmatrix} \delta_\mu{}^{\mu'} & \delta_\mu{}^{\nu'} & \delta_\mu{}^{\rho'} \\ \delta_\nu{}^{\mu'} & \delta_\nu{}^{\nu'} & \delta_\nu{}^{\rho'} \\ \delta_\rho{}^{\mu'} & \delta_\rho{}^{\nu'} & \delta_\rho{}^{\rho'} \end{vmatrix} = -\delta^{\mu'\nu'\rho'}_{\mu\nu\rho}$$

contract

$$= -\Big[\big(\delta_\mu{}^{\mu'}\delta_\nu{}^{\nu'}\delta_\rho{}^{\rho'} + \delta_\mu{}^{\nu'}\delta_\nu{}^{\rho'}\delta_\rho{}^{\mu'} + \delta_\mu{}^{\rho'}\delta_\nu{}^{\mu'}\delta_\rho{}^{\nu'}\big) - \big(\delta_\mu{}^{\rho'}\delta_\nu{}^{\nu'}\delta_\rho{}^{\mu'} + \delta_\mu{}^{\mu'}\delta_\nu{}^{\rho'}\delta_\rho{}^{\nu'} + \delta_\mu{}^{\nu'}\delta_\nu{}^{\mu'}\delta_\rho{}^{\rho'}\big)\Big]$$

$$\epsilon_{\mu\nu\rho\sigma}\,\epsilon^{\mu'\nu'\rho\sigma} = -2 \begin{vmatrix} \delta_\mu{}^{\mu'} & \delta_\mu{}^{\nu'} \\ \delta_\nu{}^{\mu'} & \delta_\nu{}^{\nu'} \end{vmatrix}$$

contracted

$$= -2\left[\delta_\mu{}^{\mu'}\delta_\nu{}^{\nu'} - \delta_\mu{}^{\nu'}\delta_\nu{}^{\mu'}\right]$$

$$\epsilon_{\mu\nu\rho\sigma}\,\epsilon^{\mu'\nu\rho\sigma} = -6\,\delta_\mu{}^{\mu'}$$

$$\epsilon_{\mu\nu\rho\sigma}\,\epsilon^{\mu\nu\rho\sigma} = -24$$