Representation theory of the Poincaré group

Poincaré group is noncompact (range of translations and boosts is infinite)

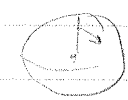
$\Longrightarrow$ unitary representations are <u>infinite dimensional</u>

Case I

$$\underline{p^2 = m^2 > 0 \,; \qquad W^2 = +m^2 s(s+1)}\,, \qquad s = \{0, \tfrac{1}{2}, 1, \tfrac{3}{2}, \ldots\}.$$

Massive particles with spin-s.

$W^0 = 0\,, \quad \vec{W} = m\vec{J}$ ← these furnish little group $SO(3)$.

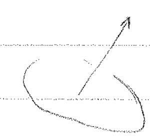


$SO(3)$

Fall into $(2s+1)$-dimensional multiplet $\quad S_3 = \{-s, \ldots, +s\}$.

Case II

$$\underline{p^2 = 0 \,; \qquad W^2 = 0} \qquad \text{if } p^\mu = (E; 0, 0, E),$$



if $p^\mu = (E; 0, 0, E)$

$$\left.\begin{array}{ll} W^0 = W^3 = E J^3 & \leftarrow \text{rotation} \\ \left.\begin{array}{l} W^1 = J^1 - k^2 \\ W^2 = -(J^2 - k^1) \end{array}\right\} & \text{translation} \end{array}\right\} \text{ Wigner's } E(2) \; \overset{\nwarrow}{\text{Euclidean}}$$

Massless particles with spin-s.

Fall into 2-dimensional helicity multiplets $\quad p\cdot J = \{-\lambda, \lambda\}$.

Case III

$$\underline{p^2 = 0 \,; \qquad W^2 = +\rho^2}$$

Massless particles with infinite number of polarization states, ρ.

(Not realized in nature).