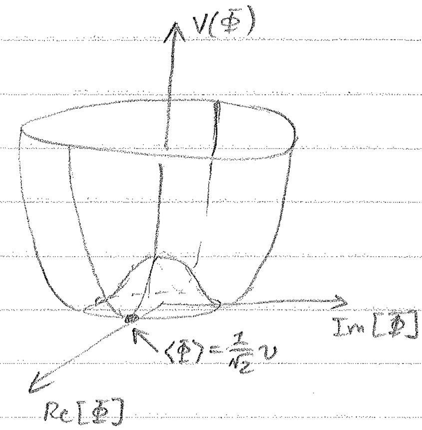
Gauge fixing in spontaneously broken gauge theories

Abelian Case

$$\mathcal{L} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - \underbrace{\left(-\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2\right)}_{V(\Phi)} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$D_\mu = \partial_\mu + ieA_\mu \quad (Q=1)$



Suppose the field Φ spontaneously acquires a non-zero vacuum expectation value — triggered by the form of $V(\Phi)$:

$$\Phi = \frac{1}{\sqrt{2}} (\underset{\substack{\uparrow \\ \text{Real part}}}{\phi_1} + i \underset{\substack{\uparrow \\ \text{Imaginary part}}}{\phi_2}) \longrightarrow \frac{1}{\sqrt{2}} (\underset{\substack{\uparrow \\ \text{VEV}}}{v} + h + i\phi)$$

$$\mathcal{L} = \Big[\underbrace{(\partial_\mu + ieA_\mu) \tfrac{1}{\sqrt{2}} (v + h + i\phi)}_{\text{expand: organize by Re \& Im}}\Big]^\dagger \Big[(\partial^\mu + ieA^\mu) \tfrac{1}{\sqrt{2}} (v + h + i\phi)\Big] - V(\Phi) - \frac{1}{4} F^2$$

$$= \frac{1}{2} \Big[(\partial_\mu h - eA_\mu \phi) + i(\partial_\mu \phi + eA_\mu h + evA_\mu)\Big]^\dagger$$
$$\Big[(\partial^\mu h - eA^\mu \phi) + i(\partial^\mu \phi + eA^\mu h + evA^\mu)\Big] - V(h,\phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

Real Part:

$$= \underbrace{\frac{1}{2} \partial_\mu h \, \partial^\mu h}_{\text{Firsts}} \underbrace{- eA_\mu (\partial^\mu h) \phi}_{\text{cross}} + \underbrace{\frac{1}{2} e^2 A_\mu A^\mu \phi^2}_{\text{lasts}}$$

Imaginary Part:

$$+ \underbrace{\frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi}_{\text{Firsts}} + \underbrace{\frac{1}{2} e^2 A_\mu A^\mu h^2}_{\text{middles}} + \underbrace{\frac{1}{2} e^2 v^2 A_\mu A^\mu}_{\text{lasts}}$$

$$+ \underbrace{eA_\mu (\partial^\mu \phi) h}_{\text{cross}} + \underbrace{e^2 v A_\mu A^\mu h}_{\text{cross}} + \underbrace{ev A_\mu (\partial^\mu \phi)}_{\text{cross}} - V(h,\phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

Organize by powers of (fluctuating fields) A, ϕ, h.

$$\boxed{\begin{aligned} \mathcal{L} = & -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \underbrace{e^2 v^2}_{\text{mass}^2} A_\mu A^\mu + \frac{1}{2} \partial_\mu h \, \partial^\mu h + \frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi + \underline{ev A_\mu (\partial^\mu \phi)} \\ & + e^2 v A_\mu A^\mu h - eA_\mu \phi \overleftrightarrow{\partial^\mu} h + \frac{1}{2} e^2 A_\mu A^\mu h^2 + \frac{1}{2} e^2 A_\mu A^\mu \phi^2 - V(h,\phi) \end{aligned}}$$

Lagrangian before gauge-fixing.

In quantizing spontaneously broken gauge theories, the R_ξ gauge fixing condition is altered to remove the $ev A_\mu (\partial^\mu \phi) \xrightarrow{\text{IBP.}} -ev(\partial . A)\phi$ term.

Following the Fadeev-Popov procedure for gauge-fixing,

$$\mathcal{L}_{GF} = \frac{-1}{2\xi}(\mathcal{F})^2$$

is added to the Lagrangian

cancel.

Renormalizable-ξ (R_ξ) gauges: $\mathcal{F} = (\partial_\mu A^\mu - \xi e v \phi)$
('t Hooft)

$$\text{Then } \mathcal{L}_{GF} = \frac{-1}{2\xi}(\partial . A - \xi e v \phi)^2$$

$$= \frac{-1}{2\xi}(\partial . A)^2 - \frac{1}{2}\xi e^2 v^2 \phi^2 + ev(\partial . A)\phi$$

Faddeev-Popov compensating ghosts:

Remark: 't Hooft's R_ξ gauge is marvelous because quantum correlation functions smoothly go from symmetry broken to symmetry restored.

Need gauge transformation rules:

$$A_\mu \to A_\mu - \frac{1}{e}\partial_\mu \alpha$$
$$h \to h - \alpha\phi \quad + \mathcal{O}(\alpha^2)$$
$$\phi \to \phi + \alpha(v+h) \quad + \mathcal{O}(\alpha^2)$$

$$\mathcal{F}(A', \phi') = \partial_\mu\left(A^\mu - \frac{1}{e}\partial^\mu \alpha\right) - \xi e v\left(\phi + \alpha(v+h)\right)$$

$$\frac{\delta \mathcal{F}(A'(x))}{\delta \alpha(y)} = \frac{\partial \mathcal{F}}{\partial \alpha}\delta(x-y) + \frac{\partial \mathcal{F}}{\partial(\partial_\mu \alpha)}\partial_\mu \delta(x-y) + \frac{\partial \mathcal{F}}{\partial(\partial_\mu \partial_\nu \alpha)}\partial_\mu \partial_\nu \delta(x-y)$$

$$= -\xi e v(v+h)\delta(x-y) - \frac{1}{e}g^{\mu\nu}\partial_\mu \partial_\nu \delta(x-y)$$

$$= -\frac{1}{e}\left(\partial^2 + \xi e^2 v(v+h)\right)\delta(x-y)$$

$$\text{Hence } \mathcal{L}_{gh} = -\frac{1}{e}\bar{\eta}\left(\partial^2 + \xi e^2 v(v+h)\right)\eta$$

Integrate by parts in 1st term.

rescale.

$$= \partial_\mu \bar{\eta}\,\partial^\mu \eta - \xi e^2 v^2 \bar{\eta}\eta - \xi e^2 v h \bar{\eta}\eta$$

For the full perturbative Lagrangian,

<u>expand the potential:</u>

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

write $\quad \Phi^\dagger \Phi = \frac{1}{2}(v + h - i\phi)(v + h + i\phi)$

$$= \frac{1}{2}(v^2 + 2vh + h^2 + \phi^2)$$

$$V(h,\phi) = -\frac{\mu^2}{2}(v^2 + 2vh + h^2 + \phi^2) + \frac{\lambda}{4}(v^2 + 2vh + h^2 + \phi^2)^2$$

h mass: $\quad m_H^2 = -\mu^2 + 3\lambda v^2 = 2\mu^2 = 2\lambda v^2$

$$= \frac{1}{2} m_H^2 h^2 + \lambda v h^3 + \frac{\lambda}{4} h^4 - \frac{v^2}{8}$$ zero point energy

$$+ \lambda v h \phi^2 + \frac{1}{2}\lambda h^2 \phi^2 + \frac{\lambda}{4}\phi^4$$

<u>Lagrangian:</u>

$$\mathcal{L} = \left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial\cdot A)^2 + \frac{1}{2}m_A^2 A^2\right) + \left(\frac{1}{2}\partial_\mu h \partial^\mu h - \frac{1}{2}m_H^2 h^2\right)$$

$$+ \left(\frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2}\xi m_A^2 \phi^2\right) + \left(\partial_\mu \bar\eta \partial^\mu \eta - \xi m_A^2 \bar\eta \eta\right)$$ free kinetic-mass terms

$$- \lambda v h^3 - \frac{\lambda}{4}h^4 - \lambda v h \phi^2 - \frac{1}{2}\lambda h^2 \phi^2 - \frac{\lambda}{4}\phi^4 + \left(\frac{+v}{8}\right)$$ ← Potential terms.

$$+ e^2 v A_\mu A^\mu h - e A_\mu \phi \overleftrightarrow{\partial}^\mu h + \frac{1}{2}e^2 A_\mu A^\mu h^2 + \frac{1}{2}e^2 A_\mu A^\mu \phi^2$$ ← scalar-gauge interactions

$$- \underbrace{\xi e^2 v}_{m_A^2/v} h \bar\eta \eta$$ ← Ghost-higgs interactions

(no Goldstone-ghost or gauge-ghost interactions in Abelian theory).

$$m_A^2 = e^2 v^2 \qquad v = +\sqrt{\mu^2/\lambda}$$

$$m_H^2 = 2\mu^2 = 2\lambda v^2$$