Deriving the Gauge Boson Propagator in R_ξ gauge.

$$\mathcal{L} = \frac{1}{2} A_\mu \left(\partial^2 g^{\mu\nu} - \left(1-\frac{1}{\xi}\right)\partial^\mu\partial^\nu + \frac{1}{2} m^2 g^{\mu\nu}\right) A_\nu$$

$$= \frac{1}{2} A_\mu \left[(\partial^2 + m^2) g^{\mu\nu} - \left(1 - \frac{1}{\xi}\right)\partial^\mu \partial^\nu \right] A_\nu$$

$\longrightarrow$ Move to Fourier space: $\partial_\mu \to -ip_\mu$

$$\frac{1}{2} \tilde{A}_\mu \underbrace{\left[(-p^2 + m^2) g^{\mu\nu} + \left(1 - \frac{1}{\xi}\right) p^\mu p^\nu \right]}_{\tilde{\mathcal{O}}^{\mu\nu}} \tilde{A}_\nu$$

Propagator $\equiv \tilde{D}_{\mu\nu}$, obtained by $\underline{\tilde{\mathcal{O}}^{\mu\nu}(-i\tilde{D}_{\nu\rho}) = \delta^\mu{}_\rho}$ (i.e. $\tilde{D}_{\nu\rho}$ is the inverse of $\tilde{\mathcal{O}}$ times i)

Then write: (add & subtract)

$$\tilde{\mathcal{O}}^{\mu\nu} = (-p^2+m^2)\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) + \left(\cancel{-p^2} + m^2 + p^2\left(\cancel{1} - \frac{1}{\xi}\right)\right)\frac{p^\mu p^\nu}{p^2}$$

$$= (-p^2+m^2)\underbrace{\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)}_{P_T^{\mu\nu}} + \left(m^2 - \frac{p^2}{\xi}\right)\underbrace{\frac{p^\mu p^\nu}{p^2}}_{P_L^{\mu\nu}}$$

Notice that $(P_T)^{\mu\nu}(P_T)_{\nu\rho} = (P_T)^\mu{}_\rho$, $(P_L)^{\mu\nu}(P_L)_{\nu\rho} = (P_L)^\mu{}_\rho$, $(P_T)^{\mu\nu}(P_L)_{\nu\rho} = 0$.

"$P_T{}^2 = P_T$" "$P_L{}^2 = P_L$" $P_T \cdot P_L = 0$.

<u>AND</u> $P_T^{\mu\nu} + P_L^{\mu\nu} = g^{\mu\nu}$ "$P_T + P_L = 1$"

The inverse of $\mathcal{O}^{\mu\nu} = A\, P_T^{\mu\nu} + B\, P_L^{\mu\nu}$ is $-i\tilde{D}_{\nu\rho} = \frac{1}{A} P_T^{\mu\nu} + \frac{1}{B} P_L^{\mu\nu}$.

check: $\mathcal{O}^{\mu\nu}(-i\tilde{D}_{\nu\rho}) = \left(A\, P_T^{\mu\nu} + B\, P_L^{\mu\nu}\right)\left(\frac{1}{A}(P_T)_{\nu\rho} + \frac{1}{B}(P_L)_{\nu\rho}\right)$

$$= \frac{\cancel{A}}{\cancel{A}} P_T^{\mu\nu}(P_T)_{\nu\rho} + \frac{\cancel{B}}{\cancel{B}} P_L^{\mu\nu}(P_L)_{\nu\rho}$$ (cross terms vanish by $P_T \cdot P_L = 0$)

$$= (P_T)^\mu{}_\rho + (P_L)^\mu{}_\rho = \delta^\mu{}_\rho. \quad \checkmark$$

Hence, the propagator is $(\tilde{D}_{VP})$ is just like $\tilde{\bar{G}}$

$$\tilde{D}^{\mu\nu} = \frac{i}{-p^2+m^2}\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) + \frac{i}{m^2 - p^2/\xi}\,\frac{p^\mu p^\nu}{p^2}$$

$$= \frac{-i}{p^2-m^2}\,g^{\mu\nu} + \frac{i}{p^2-m^2}\,\frac{p^\mu p^\nu}{p^2} - \frac{\xi i}{p^2-\xi m^2}\,\frac{p^\mu p^\nu}{p^2} \quad \leftarrow$$

Sometimes this form is useful (valid for any $\langle\phi\rangle$)

$$= \frac{-i}{p^2-m^2}\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} + \frac{\xi(p^2-m^2)}{p^2-\xi m^2}\,\frac{p^\mu p^\nu}{p^2}\right)$$

$$= \frac{-i}{p^2-m^2}\left(g^{\mu\nu} - (1-\xi)\,\frac{p^\mu p^\nu}{p^2-\xi m^2}\right) \quad \leftarrow$$

Canonical form: (Feynman gauge + ξ-dep)

Alternate form (sometimes useful to demonstrate ξ-independence of physical quantities) – add/subtract $\frac{p^\mu p^\nu}{m^2}$.

$$\tilde{D}^{\mu\nu} = \frac{-i}{p^2-m^2}\left[g^{\mu\nu} - \frac{p^\mu p^\nu}{m^2} + \frac{p^\mu p^\nu}{m^2} - (1-\xi)\frac{p^\mu p^\nu}{p^2-\xi m^2}\right]$$

$$= \frac{-i}{p^2-m^2}\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{m^2}\right) + \frac{-ip^\mu p^\nu}{p^2-m^2}\underbrace{\left(\frac{1}{m^2} - (1-\xi)\frac{1}{p^2-\xi m^2}\right)}_{\text{combine denominators.}}$$

$$\frac{p^2 - \cancel{\xi m^2} - (1-\cancel{\xi})m^2}{m^2(p^2-\xi m^2)} = \frac{p^2-m^2}{m^2(p^2-\xi m^2)}$$

$$= \underbrace{\frac{-i}{p^2-m^2}\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{m^2}\right)}_{\text{Unitarity gauge}} + \underbrace{\frac{-i}{p^2-\xi m^2}\left(\frac{p^\mu p^\nu}{m^2}\right)}_{\xi\text{-dependent.}}.$$

[starting point of pinch technique]