Gauge Fixing in a Spontaneously Broken Gauge Theory (BRST)

– Put all scalars in the real representation $\Rightarrow$ generators are purely imaginary.

Then write $\underbrace{T^a_{Im}}_{\text{Purely imaginary}} \equiv -i \underbrace{T^a_{Re}}_{\text{Purely Real}}$ so that $\partial_\mu + igA^a_\mu T^a \longrightarrow \partial_\mu + gA^a_\mu T^a_{Re}$

Then shift the field to the vacuum: $\Phi_i(x) = \langle\phi\rangle_i + \phi_i(x)$

$$\left.\begin{aligned} \delta\phi_i(x) &= +\alpha^a T^a_{ij}(\langle\phi\rangle + \phi)_j \\ \delta\langle\phi\rangle_i &= 0. \end{aligned}\right\} \text{ gauge transformation.}$$

The convenient choice for gauge fixing is R_ξ:

$$\mathcal{F}^a = \partial^\mu A^a_\mu \underset{(+)}{-} \xi\,(gT^a\langle\phi\rangle)_i\,\phi_i$$

Under BRST transformations, we have

$$i[\hat{Q}_{BRST}, \hat{\phi}_i] = \underset{(+)}{-}\eta^a (gT^a_{Re})_{ij}\,(\langle\phi\rangle + \phi)_j \equiv \underset{(+)}{-}\eta^a (gT^a_{Re})_{ij}\,\Phi_j(x)$$

$$i[\hat{Q}_{BRST}, \hat{A}^a_\mu] = \underset{(+)}{-}D^{ab}_\mu \hat{\eta}^b$$

$$i\{\hat{Q}_{BRST}, \hat{\eta}^a\} = \underset{(-)}{+}\frac{1}{2}(gf^{abc})\,\eta^b\eta^c$$

$$i\{\hat{Q}_{BRST}, \hat{\bar{\eta}}^a\} = \hat{B}^a = \frac{1}{\xi}\left(\partial^\mu \hat{A}^a_\mu \underset{(+)}{-} \xi(gT^a_{Re}\langle\phi\rangle)_i\,\phi_i\right)$$

*use lower sign if $D_\mu = \partial_\mu - igA^a_\mu T^a$