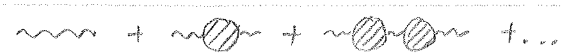
Full Propagators – general class of linear gauges

Define: $i\Pi^{\mu\nu}(p) =$  $= i\left(\Pi_A A^{\mu\nu} + \Pi_B B^{\mu\nu} + \Pi_c C^{\mu\nu} + \Pi_E E^{\mu\nu}\right)$ [1]

Then, the full propagator is

$\tilde{D}^{\mu\nu}_{\text{Full}}(p^2) =$ 

$$= \tilde{D}^{\mu\nu} + \tilde{D}^{\mu\rho}\, i\Pi_{\rho\sigma}\, \tilde{D}^{\sigma\nu} + \tilde{D}^{\mu\rho}\, i\Pi_{\rho\sigma}\, \tilde{D}^{\sigma\lambda}\, i\Pi_{\lambda\alpha}\, \tilde{D}^{\alpha\nu} + \ldots$$

$$= \tilde{D}^{\mu\rho} \sum_{n=0}^{\infty} \left(i\Pi\tilde{D}\right)_{\rho}{}^{\nu}$$

Geometric series

$$= \tilde{D}^{\mu\rho}\left(1 - i\Pi\tilde{D}\right)^{-1}{}_{\rho}{}^{\nu}$$

$$\tilde{D}^{-1\,\mu\nu}_{\text{Full}} = \left(1 - i\Pi\tilde{D}\right)^{\mu}{}_{\rho}\, \tilde{D}^{-1\,\rho\nu}$$

$$\boxed{\tilde{D}^{-1\,\mu\nu}_{\text{Full}} = \tilde{D}^{-1\,\mu\nu} - i\Pi^{\mu\nu}} \qquad [2]$$

Now, $\tilde{D}^{-1\,\mu\nu} = -i\,\hat{\mathcal{O}}^{\mu\nu}$ as in $\mathcal{L} = \frac{1}{2} A^a_\mu\, \hat{\mathcal{O}}^{\mu\nu} A^b_\nu$ (by definition)

$$= ip^2 A^{\mu\nu} + i\left(p^2 + \frac{1}{\xi}n^2\right)B^{\mu\nu} + \frac{i}{\xi}\frac{p\cdot f}{p^2}C^{\mu\nu} + \frac{i}{\xi}\frac{(p\cdot f)^2}{p^2}E^{\mu\nu} \qquad [3]$$

Plug [1] & [3] into [2]:

$$\tilde{D}^{-1\,\mu\nu}_{\text{Full}} = \underbrace{i\left(p^2 - \Pi_A\right)}_{a} A^{\mu\nu} + \underbrace{i\left(p^2 + \frac{1}{\xi}n^2 - \Pi_B\right)}_{b} B^{\mu\nu} + \underbrace{i\left(\frac{1}{\xi}\frac{p\cdot f}{p^2} - \Pi_c\right)}_{c} C^{\mu\nu} + \underbrace{i\left(\frac{1}{\xi}\frac{(p\cdot f)^2}{p^2} - \Pi_E\right)}_{e} E^{\mu\nu}$$

Invert this to obtain full propagator – follow precisely the same steps used to invert $\hat{\mathcal{O}}^{\mu\nu}$: define a, b, c & e in the same way – indicated above.

<u>Ansatz</u>: $\tilde{D}^{\mu\nu}_{\text{Full}}(p^2) = a' A^{\mu\nu} + b' B^{\mu\nu} + c' C^{\mu\nu} + e' E^{\mu\nu}$

Solve for a', b', c', e' (known from inverting $\mathcal{O}$)

Solution (borrowing previous results):

$$a' = \frac{1}{a} = \frac{-i}{p^2 - \Pi_A}$$

$$b' = \frac{e}{be - c^2 p^2 n^2} = \frac{1}{b - \frac{c^2}{e} p^2 n^2} = \frac{i}{r(p)}$$

$$c' = \frac{-c}{be - c^2 p^2 n^2} = \frac{1}{b - \frac{c^2}{e} p^2 n^2} \frac{-c}{e} = \frac{i}{r(p)} \frac{-\left(\frac{1}{\xi}\frac{p.f}{p^2} - \Pi_C\right)}{\frac{1}{\xi}\frac{(p.f)^2}{p^2} - \Pi_E}$$

$$e' = \frac{b}{be - c^2 p^2 n^2} = \frac{1}{b - \frac{c^2}{e} p^2 n^2} \frac{b}{e} = \frac{i}{r(p)} \frac{p^2 - \frac{1}{\xi} n^2 - \Pi_B}{\frac{1}{\xi}\frac{(p.f)^2}{p^2} - \Pi_E}$$

where $r(p) = i\left(b - \frac{c^2}{e} p^2 n^2\right) = -p^2 - \frac{1}{\xi} n^2 + \Pi_B + \dfrac{\left(\frac{1}{\xi}\frac{p.f}{p^2} - \Pi_C\right)^2}{\frac{1}{\xi}\frac{(p.f)^2}{p^2} - \Pi_E} p^2 n^2$

$$\Rightarrow \boxed{\tilde{D}^{\mu\nu}_{\text{Full}}(p^2) = \frac{-i}{p^2 - \Pi_A} A^{\mu\nu} + \frac{i}{r(p)} B^{\mu\nu} - \frac{i}{r(p)} \frac{(p.f) - \xi p^2 \Pi_C}{(p.f)^2 - \xi p^2 \Pi_E} C^{\mu\nu} + \frac{i}{r(p)} \frac{\xi p^2 - n^2 - \xi \Pi_B}{(p.f)^2/p^2 - \xi \Pi_E} E^{\mu\nu}}$$

<u>In CLASS OF COVARIANT GAUGES:</u> $f^\mu = -ip^\mu \rightarrow p^\mu$ (integ. by parts)

$$\Rightarrow n^\mu = 0 \qquad B^{\mu\nu} = C^{\mu\nu} = 0.$$

$$\boxed{\Pi_B(p^2) = \Pi_C(p^2) = 0}$$

$\Rightarrow r(p) = -p^2$ So coeff. of $E^{\mu\nu}$ term $= \dfrac{i}{-p^2} \dfrac{\xi p^2}{1 - \xi \Pi_E} = \dfrac{-\xi i}{p^2 - \xi \Pi_E}$

Conventionally, $\Pi_A \equiv \Pi_T$ (transverse pol. function)

$\Pi_E \equiv \Pi_L$ (longitudinal pol. function)

$$\Rightarrow \tilde{D}^{\mu\nu}_{\text{Full}}(p^2) = \frac{-i}{p^2 - \Pi_T(p^2)}\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) - \frac{\xi i}{p^2 - \xi \Pi_L(p^2)} \frac{p^\mu p^\nu}{p^2}$$

In abelian gauge theories,
Ward identity $\Rightarrow$ $\Pi_T(p^2 = 0) = 0$
$\Pi_L(p^2) = 0$

In R_ξ gauges: (spontaneously broken gauge theories)

$$\partial_\mu A^{\mu a} + \xi [g T^a \langle \phi \rangle]_i \phi_i = 0 \quad \Rightarrow \quad f^\mu = -ip^\mu \to p^\mu \text{ (integ. by parts)}$$

(only effect in quadratic form for spin-1 fields)

$$\Rightarrow n^\mu = 0 \qquad B^{\mu\nu} = C^{\mu\nu} = 0$$

$$\boxed{\Pi_B(p^2) = \Pi_C(p^2) = 0}$$

From: $\mathcal{L}_{Quad} = \frac{1}{2} A^a_\mu \left(\partial^2 g^{\mu\nu} \delta^{ab} - (1 - \frac{1}{\xi}) \partial^\mu \partial^\nu \delta^{ab} + (m_A^2)^{ab} g^{\mu\nu} \right) A^b_\nu$,

$$\Rightarrow \tilde{D}^{-1}_{\mu\nu} = i(p^2 \delta^{ab} - (m_A^2)^{ab}) \underbrace{\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right)}_{A_{\mu\nu}} + i\left(\frac{p^2}{\xi} - (m_A^2)^{ab} \right) \underbrace{\frac{p_\mu p_\nu}{p^2}}_{E_{\mu\nu}}$$

see deriving R_ξ propagator.

Add $-i \Pi^{ab}_{\mu\nu} = -i \left(\Pi^{ab}_A(p^2) A_{\mu\nu} + \Pi^{ab}_E(p^2) E_{\mu\nu} \right)$

$$\Rightarrow \tilde{D}^{-1\ ab}_{Full\ \mu\nu} = i\left(p^2 \delta^{ab} - (m_A^2)^{ab} - \Pi^{ab}_A(p^2) \right) A_{\mu\nu} + i\left(\frac{p^2}{\xi} - (m_A^2)^{ab} - \Pi^{ab}_E(p^2) \right) E_{\mu\nu}$$

Can be easily inverted since $A_{\mu\nu}$ & $E_{\mu\nu}$ are orthogonal projectors:

$$\boxed{\tilde{D}^{\ ab}_{Full\ \mu\nu} = \frac{-i}{p^2 - (m_A^2)^{ab} - \Pi^{ab}_A(p^2)} \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) + \frac{-i\xi}{p^2 - \xi (m_A^2)^{ab} - \xi \Pi^{ab}_E(p^2)} \left(\frac{p_\mu p_\nu}{p^2} \right)}$$

conventionally, $\Pi_A \equiv \Pi_T$
$\Pi_E \equiv \Pi_L$

Gauge invariance $\Rightarrow$ $\Pi_T(0) = \Pi_L(0)$ (to be checked?)

Projectors for $\Pi^{\mu\nu}$, and pole masses of gauge bosons

In covariant Fermi (e.g. $\partial \cdot A = 0$ for no Higgs mechanism) and in R_ξ gauges (with Higgs mechanism),

$$i\Pi^{\mu\nu}(p^2) = i\Pi_T(p^2)\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) + i\Pi_L(p^2)\frac{p^\mu p^\nu}{p^2}$$

$$\tilde{D}^{\mu\nu}_{Full}(p^2) = \frac{-i}{p^2 - m_A^2 - \Pi_T(p^2)}\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) + \frac{-i\xi}{p^2 - \xi(m_A^2 + \Pi_L(p^2))}\frac{p^\mu p^\nu}{p^2}$$

To extract $\Pi_T(p^2)$ and $\Pi_L(p^2)$ from a rank-2 tensor, contract with projectors:

$$T^\rho{}_\sigma = \frac{1}{d-1}\left(\delta^\rho{}_\sigma - \frac{p^\rho p_\sigma}{p^2}\right) \qquad \text{transverse projector}$$

$$L^\rho{}_\sigma = \frac{p^\rho p_\sigma}{p^2} \qquad \text{longitudinal projector}$$

and take trace:

$\boxed{\text{TRANSVERSE}}$

$$i\Pi_{\mu\nu}(p^2)\, T^\nu{}_\rho = i\Pi_T(p^2)\, T_{\mu\rho}$$

trace:

$$i\Pi_{\mu\nu} T^{\nu\mu} = i\Pi_T(p^2)\underbrace{T_\mu{}^\mu}_{1}$$

$$\boxed{\Pi_{\mu\nu} T^{\nu\mu} = \Pi_T(p^2)}$$

$\boxed{\text{LONGITUDINAL}}$

$$i\Pi_{\mu\nu}(p^2)\, L^\nu{}_\rho = i\Pi_L(p^2)\, L^\nu{}_\rho$$

trace:

$$\boxed{\Pi_{\mu\nu}(p^2)\, L^{\nu\mu} = \Pi_L(p^2)}$$

Pole masses

The position of the pole — for 3-transverse directions — is:

photon

$$p^2 - \Pi_T(p^2)\Big|_{p^2 = m^2_{pole}} = 0$$

Z/W gauge boson:

$$p^2 - m_A^2 - \Pi_T(p^2)\Big|_{p^2 = m^2_{pole}} = 0$$