# Word identities for scalar electrodynamics

external sources.

$$Z[J_\mu, j, j^*] = \int \mathcal{D}A_\mu \mathcal{D}\phi \mathcal{D}\phi^* \, e^{i\int d^4x \left[\mathcal{L}_{SQED}[A,\phi] + J_\mu A^\mu + j\phi + j^*\phi^*\right]}$$

$$\mathcal{L}_{SQED} = \begin{pmatrix}\text{Gauge-invariant} \\ \text{terms}\end{pmatrix} - \frac{1}{2\xi}(\partial.A)^2$$

Perform infinitessimal gauge transformation as a change of integration variables:

$$\phi \to e^{i\alpha}\phi \simeq \phi + i\alpha\phi + O(\alpha)^2$$
$$\phi^* \to e^{-i\alpha}\phi^* \simeq \phi^* - \phi^* i\alpha + O(\alpha)^2$$
$$A_\mu \to A_\mu - \frac{1}{e}\partial_\mu \alpha$$

Expect no change in integration measure:

IBP twice IBP once.

$$Z[J_\mu, j, j^*] \longrightarrow \int \mathcal{D}A_\mu \mathcal{D}\phi \mathcal{D}\phi^* \, e^{i\int d^4x \left[\mathcal{L}_{SQED} + \text{sources} + \frac{1}{\xi}\frac{1}{e}(\partial.A)(\partial^2\alpha) - \frac{1}{e}J_\mu \partial^\mu \alpha + i\alpha j\phi - i\alpha j^*\phi^* + O(\alpha^2)\right]}$$

$$= \int \mathcal{D}A_\mu \mathcal{D}\phi\mathcal{D}\phi^* \left[1 + \int d^4x\, \alpha(x)\left(\frac{1}{\xi}\frac{1}{e}\partial^2\partial.A + \frac{1}{e}\partial.J + ij\phi - ij^*\phi^*\right)\right] \times e^{i\int d^4x\left[\mathcal{L}_{SQED} + \text{sources}\right]}$$

$$0 = \delta Z = \int \mathcal{D}A_\mu \mathcal{D}\phi \mathcal{D}\phi^* \int d^4x\, \alpha(x)\left(\frac{1}{\xi e}\partial^2\partial.A + \frac{1}{e}\partial.J + ij\phi - ij^*\phi^*\right) e^{i\int d^4x\left[\mathcal{L}_{SQED} + \text{sources}\right]}$$

Replace: $A_\mu \to \frac{\delta}{i\delta J^\mu}$ $\quad \phi \to \frac{\delta}{i\delta j}$ $\quad \phi^* \to \frac{\delta}{i\delta j^*}$

Multiply through by e, divide by Z

True for any gauge function $\alpha(x)$

$$\boxed{0 = \left[\frac{1}{\xi}\partial^2\partial^\mu \frac{\delta}{i\delta J^\mu} + \partial_\mu J^\mu + ie\left(j\frac{\delta}{i\delta j} - j^*\frac{\delta}{i\delta j^*}\right)\right]\frac{Z[J_\mu, j, j^*]}{Z[0]}}$$

Ward identity for connected Green's functions:

Write $Z[J, j, j^*] = Z[0]\, e^{iW[J,j,j^*]}$

$$\Rightarrow \boxed{0 = \partial_\cdot J + i\left[\frac{1}{\xi}\partial^2\partial_\mu \frac{\delta}{i\,\delta J_\mu} + ie\left(j\frac{\delta}{i\delta j} - j^*\frac{\delta}{i\delta j^*}\right)\right] W[J, \eta, \bar\eta]}$$

Ward identity for 1PI vertices

$$W[J, j, j^*] = \Gamma[A, \phi, \phi^*] + \int d^4x\, (J_\mu A^\mu + j\phi + j^*\phi^*)$$

Identities: $\frac{\delta W}{\delta J_\mu} = A^\mu \qquad \frac{\delta W}{\delta j} = \phi \qquad \frac{\delta W}{\delta j^*} = \phi^*$

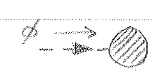
$$J^\mu = \frac{-\delta\Gamma}{\delta A_\mu} \qquad j = \frac{-\delta\Gamma}{\delta\phi} \qquad j^* = \frac{-\delta\Gamma}{\delta\phi^*}$$

$$0 = -\partial_\mu\frac{\delta\Gamma}{\delta A_\mu} + i\left[\frac{1}{\xi}\partial^2\partial_\mu \frac{1}{i}A^\mu + \cancel{i}e\left(\frac{-\delta\Gamma}{\delta\phi}\frac{1}{\cancel{i}}\phi - \frac{-\delta\Gamma}{\delta\phi^*}\frac{1}{\cancel{i}}\phi^*\right)\right]$$

$$\boxed{0 = -\partial_\mu\frac{\delta\Gamma}{\delta A_\mu} + \frac{1}{\xi}\partial^2\partial_\mu A^\mu + ie\left(\frac{\delta\Gamma}{\delta\phi^*}\phi^* - \frac{\delta\Gamma}{\delta\phi}\phi\right)}$$

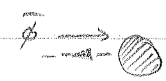
Careful with complex-valued fields:

$$\langle\hat\phi^\dagger\rangle = \left.\frac{\delta\Gamma}{\delta\phi^*}\right|_{\phi,\phi^*=0} =$$



creates particle

$$\langle\hat\phi\rangle = \left.\frac{\delta\Gamma}{\delta\phi}\right|_{\phi,\phi^*=0} =$$



creates antiparticle