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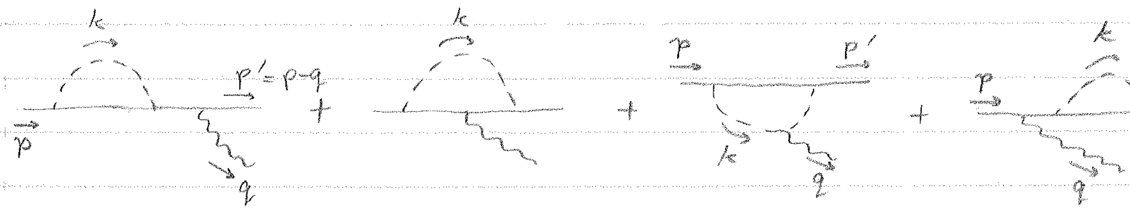
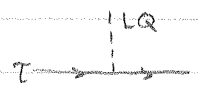
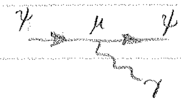
Checking Ward Identity for $\tau \to e\gamma$ (massless quarks & electron)



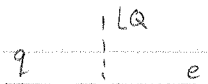
Feynman Rules (internal scalars are leptoquarks)



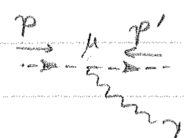
$= i\lambda \hat{P}_R$



$= -ieQ_\psi \gamma^\mu$



$= i\lambda \hat{P}_L$



$= -ieQ_{LQ}(p-p')^\mu$

Dirac Eqns

$\bar{u}_e(p-q)\hat{P}_L(\not{p}-\not{q}) = 0$

$(\not{p}-m_\tau)u_\tau(p) = 0$

$$
\begin{aligned}
i\mathcal{M}^\mu = {} & \mu^{2\epsilon}\int\frac{d^dk}{(2\pi)^d}\,\bar{u}_e(p-q)(-ieQ_\ell\gamma^\mu)\frac{i}{\not{p}}(i\lambda\hat{P}_L)\frac{i}{\not{p}-\not{k}}\,i\lambda\hat{P}_R\,u_\tau(p)\frac{i}{k^2-M_{LQ}^2} \\
& + \mu^{2\epsilon}\int\frac{d^dk}{(2\pi)^d}\,\bar{u}_e(p-q)(i\lambda\hat{P}_L)\frac{i}{\not{p}-\not{k}-\not{q}}(-ieQ_q\gamma^\mu)\frac{i}{\not{p}-\not{k}}(i\lambda\hat{P}_R)\frac{i}{k^2-M_{LQ}^2}\,u_\tau(p) \\
& + \mu^{2\epsilon}\int\frac{d^dk}{(2\pi)^d}\,\bar{u}_e(p-q)(i\lambda\hat{P}_L)\frac{i}{(k-q)^2-M^2}\left[-ieQ_{LQ}(2k^\mu-q^\mu)\right]\frac{i}{k^2-M^2}\,\frac{i}{\not{p}-\not{k}}(i\lambda\hat{P}_R)u_\tau(p) \\
& + \mu^{2\epsilon}\int\frac{d^dk}{(2\pi)^d}\,\bar{u}_e(p-q)(i\lambda\hat{P}_L)\frac{i}{\not{p}-\not{k}-\not{q}}(i\lambda\hat{P}_R)\frac{i}{\not{p}-\not{q}-m_\tau}(-ieQ_\ell\gamma^\mu)\,u_\tau(p)\frac{i}{k^2-M_{LQ}^2}
\end{aligned}
$$

In each term, push $\hat{P}_L$ & $\hat{P}_R$ as far left as possible, factor $-\lambda^2 e(i)^6$ in front.
To check Ward identity for electromagnetism, dot $\mathcal{M}^\mu$ with q_μ.

$$
\begin{aligned}
iq\cdot\mathcal{M} = \lambda^2 e\mu^{2\epsilon}\int\frac{d^dk}{(2\pi)^d}\Big[& Q_\ell\,\bar{u}_e(p-q)\,\hat{P}_L\,\not{q}\,\frac{1}{\not{p}}\,\frac{1}{\not{p}-\not{k}}\,u_\tau(p)\frac{1}{k^2-M_{LQ}^2} \\
& + Q_q\,\bar{u}_e(p-q)\,\hat{P}_L\frac{1}{\not{p}-\not{k}-\not{q}}\,\not{q}\,\frac{1}{\not{p}-\not{k}}\,u_\tau(p)\frac{1}{k^2-M_{LQ}^2} \\
& + Q_{LQ}\,\bar{u}_e(p-q)\,\hat{P}_L\frac{1}{(k-q)^2-M^2}(2k\cdot q-q^2)\frac{1}{k^2-M_{LQ}^2}\,\frac{1}{\not{p}-\not{k}}\,u_\tau(p) \\
& + Q_\ell\,\bar{u}_e(p-q)\,\hat{P}_L\frac{1}{\not{p}-\not{k}-\not{q}}\,\frac{1}{\not{p}-\not{q}-m_\tau}\,\not{q}\,u_\tau(p)\frac{1}{k^2-M_{LQ}^2}\Big]
\end{aligned}
$$

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In first term, write $\not{q} = -(\not{p} - \not{q}) + \not{p}$

In 2nd term, $\not{q} = -(\not{p} - \not{k} - \not{q}) + (\not{p} - \not{k})$

In 3rd term, $2k.q - q^2 = -(k^2 - 2k.q + q^2 - M^2) + (k^2 - M^2) = -((k-q)^2 - M^2) + (k^2 - M^2)$

In 4th term, $\not{q} = -(\not{p} - \not{q} - m_\tau) + (\not{p} - m_\tau)$

$$
\begin{aligned}
i q.M = \lambda^2 e \mu^{2\epsilon} \int \frac{d^dk}{(2\pi)^d} \Big[& - Q_\ell\, \bar{u}_e(p-q) \hat{P}_L \underbrace{(\not{p} - \not{q})}_{=0 \text{ (Dirac eqn)}} \frac{1}{\not{p}} \frac{1}{\not{p} - \not{k}} u_\tau(p) \frac{1}{k^2 - M_{LQ}^2} \\
& + Q_\ell\, \bar{u}_e(p-q) \hat{P}_L \not{p} \frac{1}{\not{p}} \frac{1}{\not{p} - \not{k}} u_\tau(p) \frac{1}{k^2 - M_{LQ}^2} \\
& - Q_q\, \bar{u}_e(p-q) \hat{P}_L \frac{1}{\not{p} - \not{k} - \not{q}} (\not{p} - \not{k} - \not{q}) \frac{1}{\not{p} - \not{k}} u_\tau(p) \frac{1}{k^2 - M_{LQ}^2} \\
& + Q_q\, \bar{u}_e(p-q) \hat{P}_L \frac{1}{\not{p} - \not{k} - \not{q}} (\not{p} - \not{k}) \frac{1}{\not{p} - \not{k}} u_\tau(p) \frac{1}{k^2 - M_{LQ}^2} \\
& - Q_{LQ}\, \bar{u}_e(p-q) \hat{P}_L \frac{1}{(k-q)^2 - M_{LQ}^2} ((k-q)^2 - M_{LQ}^2) \frac{1}{k^2 - M_{LQ}^2} \frac{1}{\not{p} - \not{k}} u_\tau(p) \\
& + Q_{LQ}\, \bar{u}_e(p-q) \hat{P}_L \frac{1}{(k-q)^2 - M_{LQ}^2} (k^2 - M_{LQ}^2) \frac{1}{k^2 - M_{LQ}^2} \frac{1}{\not{p} - \not{k}} u_\tau(p) \\
& - Q_\ell\, \bar{u}_e(p-q) \hat{P}_L \frac{1}{\not{p} - \not{k} - \not{q}} \frac{1}{\not{p} - \not{q} - m_\tau} (\not{p} - \not{q} - m_\tau) u_\tau(p) \frac{1}{k^2 - M_{LQ}^2} \\
& + Q_{LQ}\, \bar{u}_e(p-q) \hat{P}_L \frac{1}{\not{p} - \not{k} - \not{q}} \frac{1}{\not{p} - \not{q} - m_\tau} \underbrace{(\not{p} - m_\tau) u_\tau(p)}_{=0 \text{ (Dirac eqn)}} \frac{1}{k^2 - M_{LQ}^2} \Big]
\end{aligned}
$$

In this term, perform a shift $k \to k+q$ as a change of int. variables $\longrightarrow$ (6th term)

3rd & 5th terms combine, 4th & 6th terms combine.

$$
\begin{aligned}
i q.M = \lambda^2 e \mu^{2\epsilon} \int \frac{d^dk}{(2\pi)^d} \Big[& Q_\ell\, \bar{u}_e(p-q) \hat{P}_L \frac{1}{\not{p} - \not{k}} u_\tau(p) \frac{1}{k^2 - M_{LQ}^2} \\
& - (Q_q + Q_{LQ})\, \bar{u}_e(p-q) \hat{P}_L \frac{1}{\not{p} - \not{k}} u_\tau(p) \frac{1}{k^2 - M_{LQ}^2} \\
& + (Q_q + Q_{LQ})\, \bar{u}_e(p-q) \hat{P}_L \frac{1}{\not{p} - \not{k} - \not{q}} u_\tau(p) \frac{1}{k^2 - M_{LQ}^2} \\
& - Q_\ell\, \bar{u}_e(p-q) \hat{P}_L \frac{1}{\not{p} - \not{k} - \not{q}} u_\tau(p) \frac{1}{k^2 - M_{LQ}^2} \Big]
\end{aligned}
$$

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Terms combine further:

$$iq \cdot \mathcal{M} = \lambda^2 e \underbrace{\left(Q_\ell - Q_q - Q_{LQ}\right)} \mu^{2\epsilon} \int \frac{d^dk}{(2\pi)^d} \, \bar{u}_e(p-q) \hat{P}_L \left(\frac{1}{\not{p} - \not{k}} - \frac{1}{\not{p} - \not{k} - \not{q}} \right) u_\tau(p) \frac{1}{k^2 - M_{LQ}^2}$$

If this vanishes, the Word identity is satisfied.