Connection of axial anomaly to Yang-Mills BPST instanton

– nonconcervation of fermion number.

Start with the (Euclideanized) ABJ anomaly equation:

$$\left(\partial_\mu J_\mu^{Axial}\right)_E = \frac{\partial \rho}{\partial \tau} + \vec{\nabla}\cdot\vec{J} = \frac{-g^2}{8\pi^2}\,Tr\left[F_{\mu\nu}\tilde{F}_{\mu\nu}\right] = \frac{-g^2}{16\pi^2} F^a_{\mu\nu}\tilde{F}^{\mu\nu a}$$

Integrate of 4-volume:

$$\int_{-\infty}^{\infty} d\tau\, d^3x \left(\frac{\partial \rho_A}{\partial \tau} + \vec{\nabla}\cdot\vec{J}_A\right) = -2 \underbrace{\int d^4x_E \frac{g^2}{16\pi^2}\,Tr\left[F_{\mu\nu}\tilde{F}_{\mu\nu}\right]}_{\nu \text{ (winding number of instanton)}}$$

First term: $\rightarrow \int_{-\infty}^{\infty} d\tau \frac{\partial Q_A}{\partial \tau} = \Delta Q_A$

Second term: $\rightarrow 0$ (vanishing currents at spatial infinity)

$$\Delta Q_A = -2\nu$$

or

$$\underline{\Delta n_R - \Delta n_L = -2\nu}$$

A $\nu = 1$ instanton event leads to $\Delta Q_A = -2 \Rightarrow$ loss of right-chiral fermion gain of left-chiral fermion.

A $\nu = -1$ (anti-instanton) event leads to $\Delta Q_A = +2 \Rightarrow$ loss of left-chiral fermion gain of right-chiral fermion.